Chapter 9: Engineering Economic Analysis

WHAT YOU WILL LEARN

- That if money is invested, it grows
- That there are different types of interest
- How to do interest and investment calculations
- That there are different types of cash flow diagrams, which can be used to represent financial transactions
- How to represent financial transactions on a cash flow diagram
- How to include taxation
- What depreciation is, how to calculate it, and how to include it in financial calculations
- What inflation is, and how it is related to interest

The goal of any manufacturing company is to make money. This is realized by producing products with a high market value from raw materials with a low market value. The companies in the chemical process industry produce high-value chemicals from low-value raw materials.

In the previous chapters, a process flow diagram (PFD) (Chapter 1), an estimate of the capital cost (Chapter 7), and an estimate of operating costs (Chapter 8) were provided for the production of benzene. From this material, an economic evaluation can be carried out to determine

1. Whether the process generates money
2. Whether the process is attractive compared with other processes (such as those for the production of ethylbenzene, ethylene oxide, formalin, and so on, given in Appendix B)

In the next two chapters, the necessary background to perform this economic analysis is provided.

The principles of economic analysis are covered in this chapter. The material presented covers all of the major topics required for completion of the Fundamentals of Engineering (FE) examination. This is the first requirement for becoming a registered professional engineer in the United States.

It is important for you, the graduating student, to understand the principles presented in this chapter at the beginning of your professional career in order to manage your money skillfully. As a result, discussions and examples of personal money management are integrated throughout the chapter.

The evaluation of profitability and comparison of alternatives for proposed projects are covered in Chapter 10.

9.1 INVESTMENTS AND THE TIME VALUE OF MONEY

The ability to profit from investing money is the key to our economic system. In this text, investment in terms of personal financing is introduced and then applied to chemical process economics.

There are various ways to distribute personal income. The first priority is to maintain a basic (no-frills) standard of living. This includes necessary food, clothing, housing, transportation, and expenses such as taxes imposed by the government. The remaining money, termed *discretionary money*, can then be distributed. It is wise to distribute this money in a manner that will realize both your short-term and long-term goals.

Generally, there are two classifications for spending discretionary money:

1. Consume money as received. This provides immediate personal gratification and/or satisfaction. Most people experience this use for money early in life.
2. Retain money for future consumption. This is money put aside to meet future needs. These may result from hard-to-predict causes such as sickness and job layoffs or from a more predictable need for long-term retirement income. It is unlikely that you have considered these types of financial needs and you probably have little experience in investing to secure a comfortable lifestyle after you stop working.

There are two approaches to setting money aside for use at a later date:

- **Simple savings:** Put money in a safety deposit box, sugar bowl, or other such container.
- **Investments:** Put money into an investment.

These two approaches are considered in Example 9.1.

Example 9.1

Upon graduation, you start your first job at $80,000/y. You decide to set aside 10%, or $8000/y, for retirement in 40 years' time, and you assume that you will live 20 years after retiring. You have been offered an investment that will pay you $106,667/y during your retirement years for the money you invest.

1. How much money would you have per year in retirement if you had saved the money, but not invested it, until retirement?
2. How does this compare with the investment plan offered?
3. How much money was produced from the investment?

Solution

1. **Money saved:** ($8000) (40) = $320,000
   Income during retirement: $320,000/20 = $16,000/y
2. **Comparison:** (Income from savings)/(Income from investments) = $16,000/$106,667 = 0.15
3. Money Produced = Money Received − Money Invested = ($106,667)(20) − $320,000 = $1,813,340

The value of the investment is clear. The income in retirement from savings amounts to only 15% of the investment income. The amount of money provided during retirement, by setting $320,000 aside, was almost $2 million. It will be shown later that this high return on investment resulted from two factors: the long time period for the investment and the interest rate earned on the savings.

Money, when invested, makes money.

The term *investment* will now be defined.

An **investment** is an agreement between two parties, whereby one party, the **investor,** provides money, P, to a second party, the **producer,** with the expectation that the producer will return money, F, to the investor at some future specified date, where $F > P$. The terms used in describing the investment are

P: Principal or Present Value

F: Future Value

n: Years between F and P

The amount of money earned from the investment is

$$E = F - P \quad (9.1)$$

The yearly earnings rate is

$$i_s = \frac{E}{Pn} = \frac{(F-P)}{Pn} \quad (9.2)$$

where i_s is termed the simple interest rate.

Equation (9.2) rearranges to

$$\frac{F}{P} = (1 + ni_s)\text{or, in general, } \frac{F}{P} = f(n, i) \quad (9.3)$$

Example 9.2 illustrates this concept.

Example 9.2

You decide to put $1000 into a bank that offers a special rate if left in for two years. After two years you will be able to withdraw $1150.

1. Who is the producer?
2. Who is the investor?
3. What are the values of P, F, i_s, and n?

Solution

1. **Producer:** The bank has to produce $150.00 in interest after two years.
2. **Investor:** You invest $1000 in an account at the beginning of the two-year period.
3. P = $1000 (given)

F = \$1150 (given)
n = 2 years (given)

From Equation (9.2),
i_s = (\$1150 – \$1000)/(\$1000)/(2) = 0.075 or 7.5% per year

In Example 9.2, you were the investor and invested in the bank. The bank was the “money producer” and had to return to you more dollars (\$1150) than you invested (\$1000). This bank transaction is an investment commonly termed as *savings*. In the reverse situation, termed *loan*, the bank becomes the investor. You must produce money during the time of the investment.

Equations (9.1) through (9.3) apply to a single transaction between the investor and the producer that covers *n* years and uses simple interest. There are other investment schedules and interest formulations in practice; these will be covered later in this chapter.

Figure 9.1 illustrates a possible arrangement to provide the funds necessary to build a new chemical plant such as the one introduced in the narrative in Chapter 1.

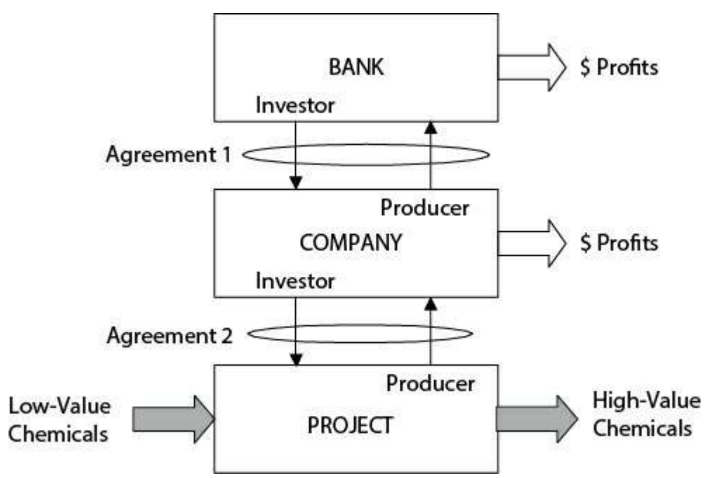

Figure 9.1 A Typical Financing Scheme for a Chemical Plant

In this arrangement, a bank invests in a company, which in turn invests in a project to produce a chemical. There are two agreements in this project (see Figure 9.1).

1. The bank is an investor and the company is the producer.
2. The company is an investor and the project is the producer.

In this illustration, all the money produced in the project is sent to the company. The company pays its investor, the bank, and draws off the rest as profit. The bank also makes a profit from its investment loan to the company. The project is the source of money to provide profits to both the company and the bank. The project converts a low-value, raw chemical into chemicals of higher value. Without the investor, the plant would not be built, and without the plant, there would be no profits for either the company or the bank.

Money is a measure of the value of products and services.

The value of a chemical material is the price it can be exchanged for in dollars. Investments may be made in units other than dollars, such as stocks, bonds, grain, oil, or gold. This is often called value, or value added, in describing investments. The term *value* is a general one and, in this case, may be assigned a dollar figure for economic calculations.

Figure 9.1 shows that all profits were produced from an operating plant. The role of engineers in our economy should be clear. This is to ensure efficient production of high-value products, including current as well as new and improved products.

In almost all cases, the economic analysis of processes will be made from the point of view of the company as the investor in a project. The project may be the construction of a new plant or a modification to an existing plant.

Consider the decisions involved in the investment in a new plant (the project) from the point of view of the company. The company must invest the money to build the plant before any income resulting from production can begin. Once the plant has been built and is operational, it is expected to operate for many years. During this time, the plant produces a profit and the company receives income from its investment. It is necessary to be able to determine whether this future income is sufficiently attractive to make the investment worthwhile.

The **time value of money** refers to a concept that is fundamental to evaluating an investment. This is illustrated in Example 9.3.

Example 9.3

You estimate that in two years' time you will need $1150 in order to replace the floor covering in your kitchen. Consider two choices:

1. Wait two years to take action.
2. Invest $1000 now (assume that interest is offered by the bank at the same rate as given in Example 9.1).

What would you do (explain your answer)?

Solution

Consider investing the $1000 today because it will provide $1150 in two years. The key is that the dollar I have today is worth 15% more than a dollar I will have in two years' time.

From Example 9.3, it was concluded that today's dollar is worth more than tomorrow's dollar, because it can be invested to earn more dollars. This must not be confused with inflation, which erodes purchasing power and is discussed in Section 9.6.

Money today is worth more than money in the future.

In the upcoming sections, it will be found that when comparing capital investments made at different times, the timing of each investment must be considered.

9.2 DIFFERENT TYPES OF INTEREST

Two types of interest are used when calculating the future value of an investment. They are referred to as *simple* and *compound interest*. Simple interest calculations are rarely used today. Unless specifically noted, all interest calculations will be carried out using compound interest methods.

9.2.1 Simple Interest

In **simple interest** calculations, the amount of interest paid is based solely on the initial investment.

Interest paid in any year = Pi_s

For an investment period of n years, the total interest paid = Pi_sn

Total value of investment in n years = $F_n = P + Pi_sn$

$$F_n = P(1 + i_s n) \tag{9.4}$$

If, instead of setting the earned interest aside, it were reinvested, the total amount of interest earned would be greater. When earned interest is reinvested, the interest is referred to as **compound interest**.

9.2.2 Compound Interest

It is possible to determine the future value of an investment, F_n, after n years at an interest rate of i per year for an initial investment of P when the interest earned is reinvested each year.

1. At the start, there is the initial investment = P.
2. In year 1, Pi in interest is earned.
   For year 2, $P + Pi$ or $P(1 + i)$ is invested.
3. In year 2, $P(1 + i)i$ in interest is earned.
   For year 3, $P(1 + i) + P(1 + i)i$, or $P(1 + i)^2$ is invested.
4. In year 3, $P(1 + i)^2 i$ in interest is earned.
   For year 4, $P(1 + i)^2 + P(1 + i)^2 i$, or $P(1 + i)^3$ is invested.
5. By induction it is found that after n years the value of the investment is $P(1 + i)^n$.

Thus, for compound interest the following can be written:

$$F_n = P(1 + i)^n \tag{9.5}$$

The process can be reversed, and the question can be asked: how much would have to be invested now, P, in order to receive a certain sum, F_n, in n years' time? The solution to this problem is found by rearranging Equation (9.5):

$$P = \frac{F_n}{(1 + i)^n} \tag{9.6}$$

The use of these equations is illustrated in Examples 9.4, 9.5, and 9.6. The letters *p.a.* following the interest refers to per year (*per annum*).

Example 9.4

For an investment of $500 at an interest rate of 8% p.a. for four years, what would be the future value of this investment, assuming compound interest?

Solution

From Equation (9.5) for $P = 500$, $i = 0.08$, and $n = 4$

$F_4 = P(1 + i)^n = 500(1 + 0.08)^4 = \$680.24$

Note: Simple interest would have yielded $F_4 = 500(1 + (4)(0.08)) = \$660$ ($20.24 less).

Example 9.5

How much investment would be needed in a savings account yielding 6% interest p.a. to have $5000 in five years' time?

Solution

From Equation (9.6) using $F_5 = \$5000$, $i = 0.06$, and $n = 5$

$P = F_n/(1 + i)^n = 5000/(1.06)^5 = \$3736.29$

If $3736.29 is invested into the savings account today, the accumulated value will be $5000 in five years' time.

Example 9.6

When borrowing a sum of money (P), it is assumed that there are two loan alternatives.

1. Borrow from my local bank, which will lend money at an interest rate of 7% p.a. and pay compound interest.
2. Borrow from "Honest Sam," who offers to lend money at 7.3% p.a. using simple interest.

In both cases, the money is needed for three years. How much money would be needed in three years to pay off this loan? Consider each option separately.

Bank: From Equation (9.5) for $n = 3$ and $i = 0.07$

$F_3 = (P)(1 + 0.07)^3 = 1.225P$

Sam: From Equation (9.4) for $n = 3$ and $i = 7.3$

$F_3 = (P)(1 + (3)(0.073)) = 1.219P$

Sam stated a higher interest rate, and yet it is still preferable to borrow the money from Sam because $1.219P < 1.225P$. This is because Sam used simple interest, and the bank used compound interest.

9.2.3 Interest Rates Changing with Time

If there is an investment over a period of years and the interest rate changes each year, then the appropriate calculation for compound interest is given by

$$F_n = P\prod_{j=1}^{n}(1+i_j) = P(1+i_1)(1+i_2)\cdots(1+i_n) \tag{9.7}$$

9.3 TIME BASIS FOR COMPOUND INTEREST CALCULATIONS

In industrial practice, the length of time assumed when expressing interest rates is one year. However, sometimes terms such as 6% p.a. compounded monthly are used. In this case, the 6% is referred to as a **nominal annual interest rate**, i_{nom}, and the number of compounding periods per year is m (12 in this case). The nominal rate is not used directly in any calculations. The **actual rate** is the interest rate per compounding period, r. The relationship needed to evaluate r is

$$r = \frac{i_{nom}}{m} \tag{9.8}$$

This is illustrated in Example 9.7.

Example 9.7

For the case of 12% p.a. compounded monthly, what are m, r, and i_{nom}?

Solution

Given: m = 12 (months in a year), i_{nom} = 12% = 0.12

From Equation (9.8),

r = 0.12/12 = 0.01 (or 1% per month)

9.3.1 Effective Annual Interest Rate

An **effective annual interest rate**, i_{eff}, can be used, which allows interest calculations to be made on an annual basis and gives the same result as using the actual compounding periods. From the value of an investment after one year,

$$F_1 = P(1+i_{eff}) = P\left(1+\frac{i_{nom}}{m}\right)^m$$

which, upon rearrangement, gives

$$i_{eff} = \left(1+\frac{i_{nom}}{m}\right)^m - 1 \tag{9.9}$$

Effective annual interest rate is illustrated in Example 9.8.

Example 9.8

What is the effective annual interest rate for a nominal rate of 8% p.a. when compounded monthly?

Solution

From Equation (9.9), for i_{nom} = 0.08 and m = 12,
$i_{eff} = (1 + 0.08/12)^{12} - 1 = 0.083$(or 8.3% p.a.)

The effective annual interest rate is greater than the nominal annual rate. This indicates that the effective interest rate will continue to increase as the number of compounding periods per year increases. For the limiting case, interest is compounded continuously.

9.3.2 Continuously Compounded Interest

For the case of continuously compounded interest, Equation (9.9) must be observed as $m \to \infty$:

$$i_{eff} = \lim_{m\to\infty}\left[\left(1+\frac{i_{nom}}{m}\right)^{m} - 1\right]$$

Rewriting the left-hand side as $\lim_{m\to\infty}\left[\left\{\left(1+\frac{i_{nom}}{m}\right)^{\frac{m}{i_{nom}}}\right\}^{i_{nom}} - 1\right]$

and noting that $\lim_{n\to\infty}\left[1+\frac{x}{n}\right]^{\frac{n}{x}} = e$

it is found that for continuous compounding,

$$i_{eff} = e^{i_{nom}} - 1 \quad (9.10)$$

Equation (9.10) represents the maximum effective annual interest rate for a given nominal rate.

The method for calculating continuously compounded interest is illustrated in Example 9.9.

Example 9.9

What is the effective annual interest rate for an investment made at a nominal rate of 8% p.a. compounded continuously?

Solution

From Equation (9.10) for i_{nom} = 0.08
$i_{eff} = e^{0.08} - 1 = 0.0833$, or 8.33% p.a.

Note: It can be seen by comparison with Example 9.8 that by compounding continuously little was gained over monthly compounding.

In comparing alternatives, the effective annual rate, and not the nominal annual rate of interest, must be used.

9.4 CASH FLOW DIAGRAMS

To this point, only an investment made at a single point in time at a known interest rate has been considered, and it was shown how to evaluate the future value of this investment. More complicated transactions involve several investments and/or payments of differing amounts made at different times. For more complicated investment schemes, careful track must be kept of the amount and time of each transaction. An effective way to track these transactions is to utilize a **cash flow diagram**, or CFD. Such a diagram offers a visual representation of each investment. Figure 9.2 is the cash flow diagram for Example 9.10, which is used to introduce the basic elements of the **discrete CFD**.

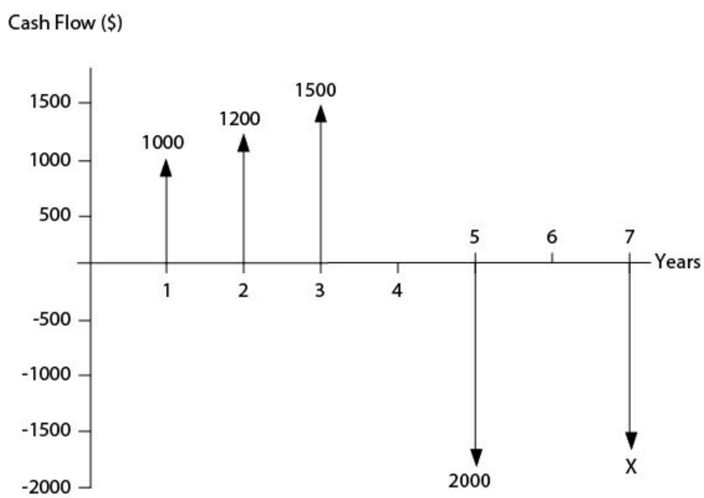

Figure 9.2 An Example of a Representative Discrete Cash Flow Diagram (CFD)

Figure 9.2 shows that cash transactions were made periodically. The values given represent payments made at the end of the year. Figure 9.2 shows that \$1000, \$1200, and \$1500 were received at the end of the first, second, and third year, respectively. In the fifth and seventh year, \$2000 and \$*X* were paid out. There were no transactions in the fourth and sixth years.

Each cash flow is represented by a vertical line, with length proportional to the cash value of the transaction. The sign convention uses a downward-pointing arrow when cash flows outward and an upward-pointing arrow representing inward cash flows. When a company invests money in a project, the company CFD shows a negative cash flow (outward flow), and the project CFD shows a positive cash flow (inward flow). Lines are placed periodically in the horizontal direction to represent the time axis. Most frequently, an analysis is performed from the point of view of the investor.

The cash flow diagram shown in Figure 9.2 can be presented in a simplified format, using the following simplifications:

1. The y-axis is not shown.
2. Units of monetary transactions are not given for every event.

In addition to the discrete CFD described above, the same information can be shown in a **cumulative CFD.** This type of CFD presents the accumulated cash flow at the end of each period.

9.4.1 Discrete Cash Flow Diagram

The discrete CFD provides a clear, unambiguous pictorial record of the value, type, and timing of each transaction occurring during the life of a project. In order to avoid making mistakes and save time, it is recommended that prior to doing any calculations, a cash flow diagram be sketched. Examples 9.10 and 9.11 illustrate the use of discrete cash flow diagrams.

Example 9.10

\$1000, \$1200, and \$1500 is borrowed from a bank (at 8% p.a. effective interest rate) at the end of years 1, 2, and 3, respectively. At the end of year 5, a payment of \$2000 is made, and at the end of year 7, the loan in paid off in full. The CFD for this exchange from the borrower's point of view (producer) is given in Figure E9.10(a).

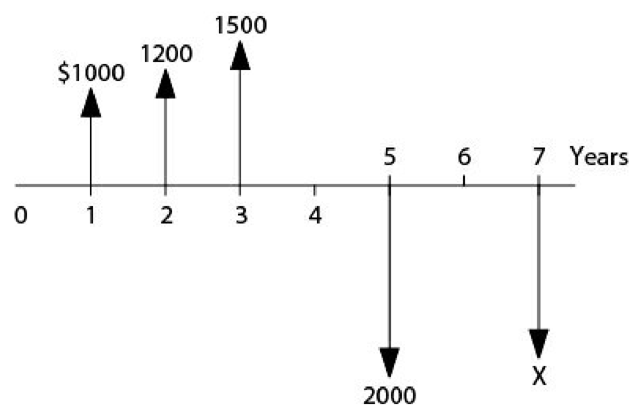

Figure E9.10a CFD for Example 9.10 from the Borrower's Perspective

Note: Figure E9.10(a) is the shorthand version of the one presented in Figure 9.2 used to introduce the CFD.

Draw a discrete cash flow diagram for the investor.

The bank represents the investor. From the investor's point of view, the initial three transactions are negative and the last two are positive.

Solution

Figure E9.10(b) represents the CFD for the bank. It is the mirror image of the one given in the problem statement.

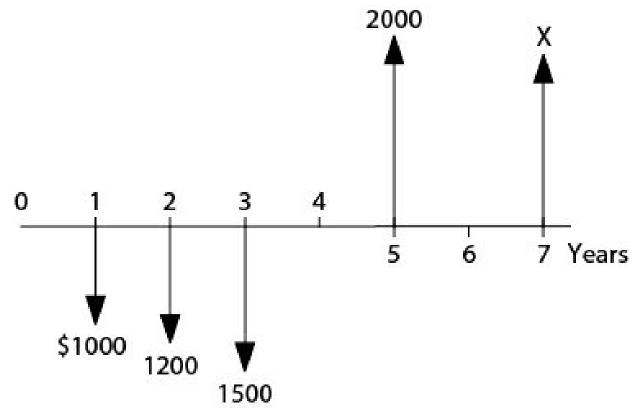

Figure E9.10b CFD for Example 9.10 from the Bank's Perspective

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The value of *X* in Example 9.10 depends on the interest rate. Its value is a direct result of the time value of money. The effect of interest rate and the calculation of the value of *X* (in Example 9.13) are determined in the next section.

Example 9.11

$10,000 is borrowed from a bank to buy a new car with 36 equal monthly payments of $320 each to repay the loan. Draw the discrete CFD for the investor in this agreement.

Solution

The bank is the investor. The discrete CFD for this investment is shown in Figure E9.11.

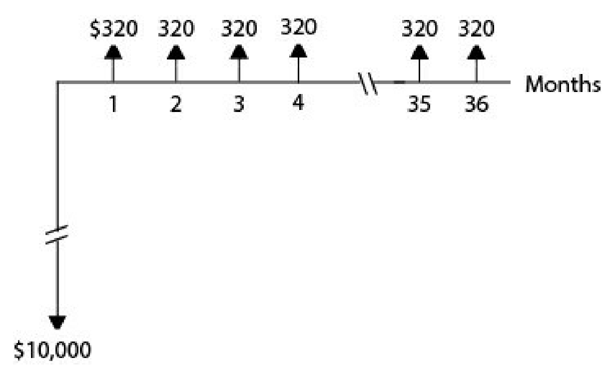


Figure E9.11 CFD for Car Loan Described in Example 9.11

Notes:

1. There is a break in both the time scale and in the investment at time = 0 (the initial investment).
2. From your point of view, the cash flow diagram would be the mirror image of the one shown.

The cash flow diagram constructed in Example 9.11 is typical of those that will be encountered throughout this text. The investment (negative cash flow) is made early in the project during design and construction, before there is an opportunity for the plant to produce product and generate money to repay the investor. In Example 9.11, payback was made in a series of equal payments over three years to repay the initial investment by the bank. In Section 9.5, how to calculate the interest rate charged by the bank in this example will be explained.

9.4.2 Cumulative Cash Flow Diagram

As the name suggests, the cumulative CFD keeps a running total of the cash flows occurring in a project. To illustrate how to construct a cumulative CFD, consider Example 9.12, which illustrates the cash flows associated with the construction and operation of a new chemical plant.

Example 9.12

The yearly cash flows estimated for a project involving the construction and operation of a chemical plant producing a new product are provided in the discrete CFD in Figure E9.12a. Using this information, construct a cumulative CFD.

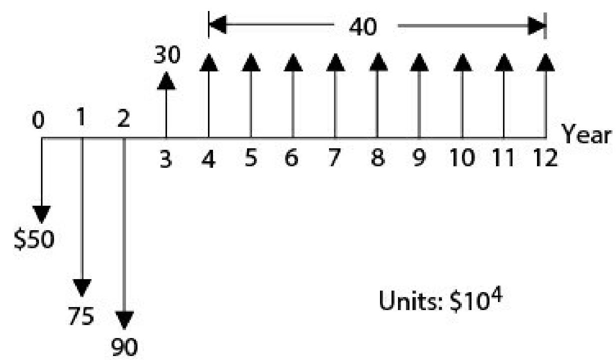


Figure E9.12a Discrete CFD for Chemical Plant Described in Example 9.12

The numbers shown in Table E9.12 were obtained from this diagram.

Table E9.12 Summary of Discrete and Cumulative Cash Flows in Example 9.12

Year	Cash Flow ($) (from Discrete CFD)	Cumulative Cash Flow (Calculated)
0	−500,000	−500,000
1	−750,000	−1,250,000
2	−900,000	−2,150,000
3	300,000	−1,850,000
4	400,000	−1,450,000
5	400,000	−1,050,000
6	400,000	−650,000
7	400,000	−250,000
8	400,000	150,000
9	400,000	550,000

10	400,000	950,000
11	400,000	1,350,000
12	400,000	1,750,000

Solution

The cumulative cash flow diagram is plotted in Figure E9.12b.

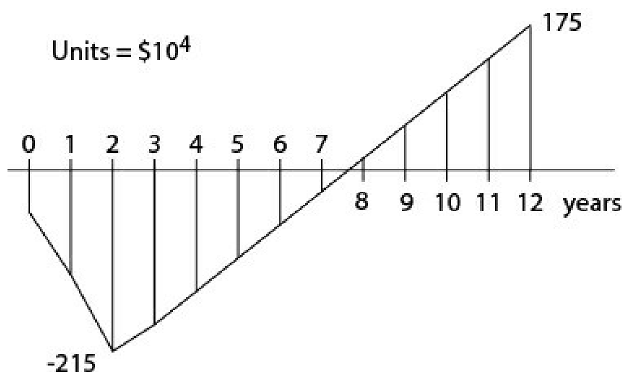


Figure E9.12b Cumulative CFD for Chemical Plant Described in Example 9.12

9.5 CALCULATIONS FROM CASH FLOW DIAGRAMS

To compare investments that take place at different times, it is necessary to account for the time value of money.

> When cash flows occur at different times, each cash flow must be brought forward (or backward) to the same point in time and then compared.

The point in time that is chosen is arbitrary. This is illustrated in Example 9.13.

Example 9.13

The CFD obtained from Example 9.10 (for the borrower) is repeated in Figure E9.13. The interest rate paid on the loan is 8% p.a.

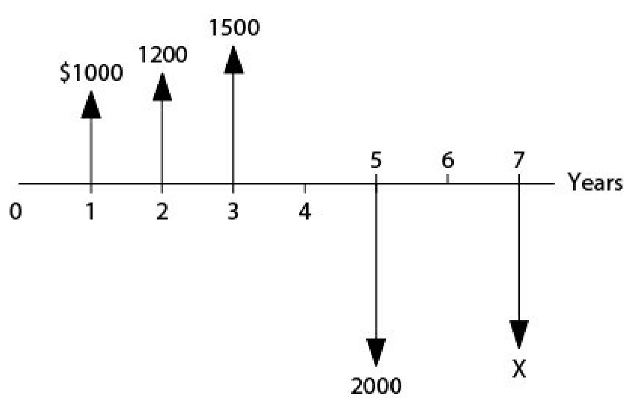


Figure E9.13 CFD for Example 9.13

In year 7, the remaining money owed on the loan is paid off.

1. Determine the amount, X, of the final payment.
2. Compare the value of X with the value that would be owed if there were no interest paid on the loan.

Solution

With the final payment at the end of year 7, no money is owed on the loan. If all the positive and negative cash flows adjusted for the time of the transactions are summed, this adjusted sum must equal zero.

The date of the final payment is selected as the base time.

1. From Equation (9.5) for $i = 0.08$ the following is obtained:

   For withdrawals:

   \$1000 end of year 1: $F_6 = (\$1000)(1 + 0.08)^6 = \$1586.87$
   \$1200 end of year 2: $F_5 = (\$1200)(1 + 0.08)^5 = \$1763.19$
   \$1500 end of year 3: $F_4 = (\$1500)(1 + 0.08)^4 = \$2040.73$
   Total withdrawals = \$5390.79

   For repayments:

   \$2000 end of year 5: $F_2 = -(\$2000)(1 + 0.08)^2 = -\$2332.80$
   \$$X$ end of year 7: $F_0 = -(\$X)(1 + 0.08)^0 = -\$X$
   Total repayments = $-\$(2332.80 + X)$

   Summing the cash flows and solving for X yields

   $0 = \$5390.79 - \$(2332.80 + X)$
   $X = \$3057.99 = \$3058$

2. For $i = 0.00$

   Withdrawals = \$1000 + \$1200 + \$1500 = \$3700
   Repayments = $-\$(2000 + X)$

   $0 = \$3700 - \$(2000 + X)$
   $X = \$1700$

Note: Because of the interest paid to the bank, the borrower repaid a total of \$1358 (\$3058 – \$1700) more than was borrowed from the bank seven years earlier.

To demonstrate that any point in time could be used as a basis, the amount repaid based on the end of year 1 can be calculated. Equation (9.6) is used, and all cash flows are moved backward in time (exponents become negative). This gives

$$0 = 1000 + \frac{1200}{1.08} + \frac{1500}{1.08^2} - \frac{2000}{1.08^4} - \frac{X}{1.08^6}$$

and solving for X yields

$$X = (1.08)^6 \left[1000 + \frac{1200}{1.08} + \frac{1500}{1.08^2} - \frac{2000}{1.08^4}\right]$$
$$= \$3058 (\text{the same answer as before!})$$

Usually, the desire is to compute investments at the start or at the end of a project, but the conclusions drawn are independent of where that comparison is made.

9.5.1 Annuities—A Uniform Series of Cash Transactions

Problems are often encountered involving a series of uniform cash transactions, each of value *A*, taking place at the end of each year for *n* consecutive years. This pattern is called an **annuity**, and the discrete CFD for an annuity is shown in Figure 9.3.

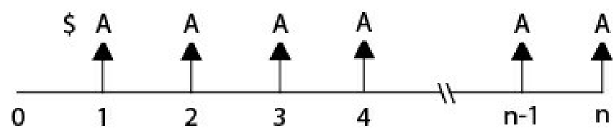


Figure 9.3 A Cash Flow Diagram for an Annuity Transaction

To avoid the need to do a year-by-year analysis like the one in Example 9.13, an equation can be developed to provide the future value of an annuity.

The future value of an annuity at the end of time period *n* is found by bringing each of the investments forward to time *n*, as was done in Example 9.13.

$$F_n = A(1+i)^{n-1} + A(1+i)^{n-2} + A(1+i)^{n-3} + \cdots + A$$

This equation is a geometric series of the form $a, ar, ar^2, \ldots, ar^{n-1}$ with sum $S_n = F_n$.

$$S_n = a\left[\frac{r^n - 1}{r - 1}\right]$$

For the present case, $a = A$; $r = 1 + i$; $n = n$. Therefore,

$$F_n = A\left[\frac{(1+i)^n - 1}{i}\right] \tag{9.11}$$

It is important to notice that Equation (9.11) is correct when the annuity starts at the end of the *first* time period and not at time zero. In the next section, a shorthand notation is provided that will be useful in CFD calculations.

9.5.2 Discount Factors

The shorthand notation for the future value of an annuity starts with Equation (9.11). The term F_n is shortened by simply calling it *F*, and then dividing through by *A* yields

$$F/A = [(1+i)^n - 1]/i = f(i, n)$$

This ratio of *F*/*A* is a function of *i* and *n*—that is, *f*(*i*,*n*). It can be evaluated when both the interest rate, *i*, and the time duration, *n*, are known. The value of *f*(*i*,*n*) is referred to as a **discount factor.** If either *A* or *F* is known, the remaining unknown can be evaluated.

In general terms, a discount factor is designated as

$$\text{Discount factor for} X/Y = (X/Y, i, n) = f(i, n)$$

Discount factors represent simple ratios and can be multiplied or divided by each other to give additional discount factors. For example, assume that the present worth, P, of an annuity, A, must be known—that is, the discount factor for P/A —but the needed equation is not available. The only available formula containing the annuity term, A, is the one for F/A derived above. The future value, F, can be eliminated, and the present value, P, introduced by multiplying by the ratio of P/F, from Equation 9.6.

$$\begin{aligned}\text{Discount factor for} P/A &= (P/A, i, n) \\ &= (F/A, i, n)\,(P/F, i, n)\end{aligned}$$

Substituting for F/A and P/F gives

$$P/A = \frac{(1+i)^n - 1}{i}\frac{1}{(1+i)^n}$$

$$= \frac{(1+i)^n - 1}{1}\frac{1}{i(1+i)^n} = (P/A, i, n)$$

Table 9.1 lists the most frequently used discount factors in this text with their common names and corresponding formulae.

Table 9.1 Commonly Used Factors for Cash Flow Diagram Calculations

Conversion	Symbol	Common Name	Eq. No.	Formula
P to F	$(F/P, i, n)$	Single Payment Compound Amount Factor	(9.5)	$(1 + i)^n$
F to P	$(P/F, i, n)$	Single Payment Present Worth Factor	(9.6)	$\frac{1}{(1+i)^n}$
A to F	$(F/A, i, n)$	Uniform Series Compound Amount Factor, Future Worth of Annuity	(9.11)	$\frac{(1+i)^n-1}{i}$
F to A	$(A/F, i, n)$	Sinking Fund Factor	(9.12)	$\frac{i}{(1+i)^n-1}$
P to A	$(A/P, i, n)$	Capital Recovery Factor	(9.13)	$\frac{i(1+i)^n}{(1+i)^n-1}$
A to P	$(P/A, i, n)$	Uniform Series Present Worth Factor, Present Worth of Annuity	(9.14)	$\frac{(1+i)^n-1}{i(1+i)^n}$

The key to performing any economic analysis is the ability to evaluate and compare equivalent investments. In order to understand that the equations presented in Table 9.1 provide a comparison of alternatives, it is suggested to replace the equal sign with the words “is equivalent to.” As an example, consider the equation given for the value of an annuity, A, needed to provide a specific future worth, F. From Table 9.1, Equation (9.11) can be expressed as

$$F \text{ (Future value) is equivalent to } \{ f(i,n) A(\text{Annuity value}) \}$$

where

$$f(i,n) = (F/A, i, n)$$

Example 9.14 illustrates a future value calculation.

Example 9.14

A lottery winner will receive $200,000/year for the next 20 years. What is the equivalent present value of the winnings if there is a secure investment opportunity providing 7.5% p.a.? What rate of return would be needed for a present value of $2.5 million?

Solution

From Table 9.1, Equation (9.14), for n = 20 and i = 0.075,

$$P = (\$200,000)\left[(1+0.075)^{20} - 1\right] / \left[(0.075)(1+0.075)^{20}\right]$$
$$P = \$2,038,900$$

A present value of $2,038,900 is equivalent to a 20-year annuity of $200,000/y when the effective interest rate is 9.5%.

To determine the interest rate needed for a present value of $2.5 million

$$\$2,500,000 = (\$200,000)\left[(1+i)^{20} - 1\right] / \left[i(1+i)^{20}\right]$$
$$i = 0.0495$$

The present value is higher for a lower interest rate. This makes sense, since it is equivalent to being able to borrow more at a lower rate for the same periodic payment.

Examples 9.15 through 9.17 illustrate how to use these discount factors and how to approach problems involving discrete CFDs.

Example 9.15

Consider Example 9.11, involving a car loan. The discrete CFD from the bank's point of view was shown.

What interest rate is the bank charging for this loan?

The agreement is to make 36 monthly payments of $320. The time selected for evaluation is the time at which the final payment is made. At this time, the loan will be fully paid off. This means that the future value of the $10,000 borrowed is equivalent to a $320 annuity over 36 payments.

$$(\$10,000)\,(F/P, i, n) = (\$320)\,(F/A, i, n)$$

Substituting the equations for the discount factors given in Table 9.1, with n = 36 months, yields

$$0 = -\,(10,000)\,(1+i)^{36} + (320)\,[(1+i)^{36} - 1]/i$$

This equation cannot be solved explicitly for i. It can be solved by plotting the value of the right-hand side of the equation shown above for various interest rates or by a numerical method. From Figure E9.15, the interest rate that gives a value of zero represents the answer. From Figure E9.15 the rate of interest is i = 0.0079.

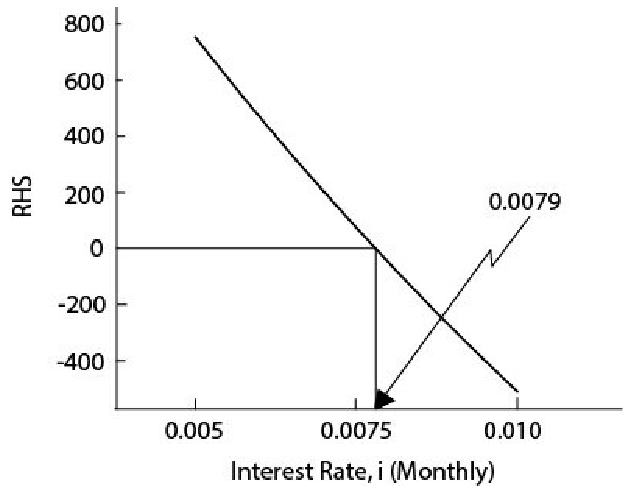


Figure E9.15 Determination of Interest Rate for Example 9.15

The nominal annual interest rate is (12)(0.00786) = 0.095 (9.5%).

Example 9.16

Money is invested in a savings account that pays a nominal interest rate of 6% p.a. compounded monthly. The account is opened with a deposit of \$1000, and then deposits of \$50 at the end of each month are made for a period of two years, followed by a monthly deposit of \$100 for the following three years. What will the value of the savings account be at the end of the five-year period?

Solution

First, a discrete CFD is drawn (Figure E9.16). Although this CFD looks rather complicated, it can be broken down into three easy subproblems:

1. The initial investment
2. The 24 monthly investments of \$50
3. The 36 monthly investments of \$100

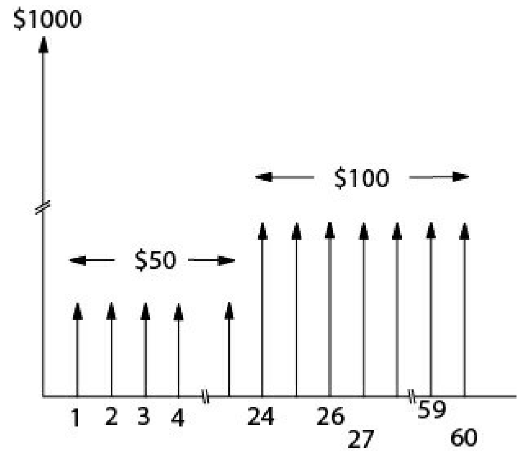


Figure E9.16 Cash Flow Diagram for Example 9.16

Each of these investments is brought forward to the end of month 60.

$$F = (\$1000)\,(F/P, 0.005, 60) \\ + (\$50)\,(F/A, 0.005, 24)\,(F/P, 0.005, 36) \\ + (\$100)\,(F/A, 0.005, 36)$$

Note: The effective monthly interest rate is 0.06/12 = 0.005.

$$F = (\$1000)\,(1.005)^{60} \\ + (\$50)\,\frac{(1.005^{24} - 1)}{0.005}(1.005)^{36} \\ + (\$100)\,\frac{(1.005^{36} - 1)}{0.005} = \$6804.16$$

There are many ways to solve most complex problems. No one method is more or less correct than another. For example, the discrete CFD could be considered to be made up of a single investment of \$1000 at the start, a \$50 monthly annuity for the next 60 months, and another \$50 annuity for the last 36 months. Evaluating the future worth of the investment gives

$$F = (\$1000)\,(F/P, 0.005, 60) \\ + (\$50)\,(F/A, 0.005, 60) + (\$50)\,(F/A, 0.005, 36)$$

$$F = (\$1000)\,(1.005)^{60} + (\$50)\,\frac{(1.005^{60} - 1)}{0.005} \\ + (\$50)\,\frac{(1.005^{36} - 1)}{0.005} = \$6804.16$$

This is the same result as before.

Example 9.17

In Example 9.1, an investment plan for retirement was introduced. It involved investing \$8000/year for 40 years leading to retirement. The plan then provided \$106,667/year for 20 years of retirement income.

1. What yearly interest rate was used in this evaluation?
2. How much money was invested in the retirement plan before withdrawals began?

Solution

1. The evaluation is performed in two steps.

   Step 1: Find the value of the $8000 annuity investment at the end of the 40 years.

   Step 2: Evaluate the interest rate of an annuity that will pay out this amount in 20 years at $106,667/y.

   Step 1: From Equation (9.11), Table 9.1, for A = $5000 and n = 40,

$$F_{40} = (A)\,(F/A, n, i) = (\$8000)\,[(1+\mathrm{i})^{40} - 1]/i$$

   Step 2: From Equation (9.14), Table 9.1, for A = $106,667 and n = 20,

$$P = (A)\,(P/A, n, i) = (\$106,667)\,[(1+i)^{20} - 1]/[(i)\,(1+i)^{20}]$$

   Set $F_{40} = P$ and solve for i. From Figure E9.17, i = 0.060.

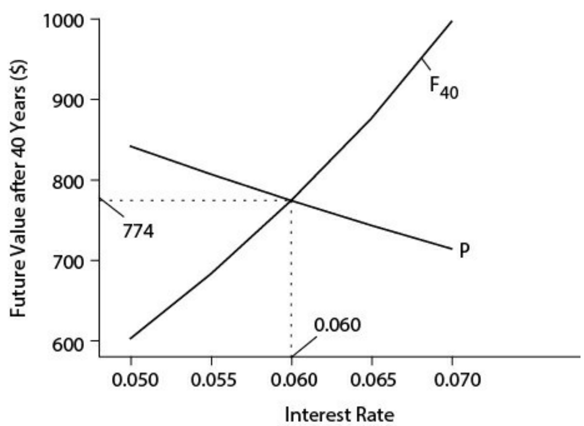


Figure E9.17 Determination of Interest Rate for Example 9.17

2. With i = 0.060, from Figure E9.17, F_{40} = $1,227,000.

   Note: The interest rate of 6.0% p.a. represents a relatively low interest rate, involving small risk.

9.6 INFLATION

As a result of inflation, a dollar set aside (not invested) will purchase fewer goods and services in the future than the same dollar would today. In Chapters 7 and 8, it was seen that inflation of equipment, labor, and fuel costs could be tracked by the use of cost indexes. It is sometimes desirable to express these trends in terms of rates of inflation (f). This can be done using the cost indexes as follows:

$$CEPCI\,(j+n) = (1+f)^{n}\,CEPCI(j) \tag{9.15}$$

where n = time span in years

f = average inflation rate over the time span

j = arbitrary year

The use of Equation (9.15) to estimate the inflation rate is illustrated in Example 9.18.

Example 9.18

What was the average rate of inflation for the costs associated with building a chemical plant over the following periods?

1. 2001 through 2007
2. 2007 through 2015

Solution

From Table 7.4, the values of the Chemical Engineering Plant Cost Index (*CEPCI*) are

CEPCI (2001) = 394

CEPCI (2007) = 500

CEPCI (2015) = 557

Equation (9.15) yields

1. $500 = 394\,(1 + f)^6$
   $f = 0.049$ (4.9% p.a.)
2. $557 = 500\,(1 + f)^6$
   $f = 0.018$ (1.8% p.a.)

To understand inflation, it is necessary to distinguish between cash and the purchasing power (for the purchase of goods and services) of cash. Inflation decreases this purchasing power with time. All the previous discussions on A, P, and F are given in terms of cash and not in terms of the relative purchasing power of this cash. The term F' is introduced, which represents the purchasing power of future cash. This purchasing power can be estimated using Equation (9.16):

$$F' = \frac{F}{(1+f)^n} \tag{9.16}$$

Substituting the equation for F in terms of P, from Equation (9.5), gives

$$F' = P\frac{(1+i)^n}{(1+f)^n} = P\left[\frac{1+i}{1+f}\right]^n \tag{9.17}$$

Equation (9.17) is now written in terms of an effective interest rate, i', which includes the effect of inflation:

$$F' = P(1+i')^n \tag{9.18}$$

By comparing Equation (9.18) with Equation (9.17), it is seen that i' is given by

$$i' = \frac{1+i}{1+f} - 1 = \frac{i-f}{1+f} \qquad (9.19)$$

For small values of $f < 0.05$, Equation (9.19) can be approximated by

$$i' \approx (i - f) \qquad (9.20)$$

Example 9.19 demonstrates the incorporation of inflation into a calculation.

Example 9.19

In this example, consider the effect of inflation on the purchasing power of the money set aside for retirement in Example 9.17. Previously, the amount of cash available at the time of retirement in 40 years was calculated to be $1,227,000. This provided an income of $106,667 for 20 years.

1. Assuming an annual inflation rate of 2%, what is the purchasing power of the cash available at retirement?
2. What is the purchasing power of the retirement income in the first and twentieth years of retirement?
3. How does Part (a) compare with the total annuity payments of $8000/y for 40 years?

Solution

1. Using Equation (9.16) for f = 0.02, n = 40, and F = $1,227,000,

   $F' = \$1,227,000/(1 + 0.02)^{40} = \$555,700$

2. At the end of the forty-first year (first year of retirement),
   Purchasing Power = $\$106,667/(1 + 0.02)^{41} = \$47,361/y$
   At the end of the sixtieth year (twentieth year of retirement),
   Purchasing Power = $\$106,667/(1 + 0.02)^{60} = \$32,510/y$
3. Amount invested = ($8000/y) (40 y) = $320,000, compared with $555,700

Example 9.19 reveals the consequences of inflation. It showed that the actual income received in retirement of $106,667/y had a purchasing power equivalent to between $32,510 and $47,361 at the time the initial investment was made. This does not come close to the $80,000/y base salary at that time. To increase this value, one or more of the following would have to be increased:

1. The amount invested
2. The interest rate for the investment
3. The time over which the investment was made

The effects of inflation should not be overlooked in any decisions involving investments. Because inflation is influenced by politics, future world events, and so on, it is hard to predict. In this book, inflation will not be considered directly, and cash flows will be considered to be in uninflated dollars.

9.7 DEPRECIATION OF CAPITAL INVESTMENT

When a company builds and operates a chemical process plant, the physical plant (equipment and buildings) associated with the process has a finite life. The value or worth of this physical plant decreases with time. Some of the equipment wears out and has to be replaced during the life of the plant. Even if the equipment is seldom used and is well maintained, it becomes obsolete and of little value. When the plant is closed, the plant equipment can be salvaged and sold for only a fraction of the original cost.

The cash flows associated with the purchase and installation of equipment are expenses that occur before the plant is operational. This results in a negative cash flow on a discrete CFD. When the plant is closed, equipment is salvaged, and this results in a positive cash flow at that time. The difference between these costs represents capital depreciation.

For tax purposes, the government does not allow companies to charge the full costs of the plant as a one-time expense when the plant is built. Instead, it allows only a fraction of the capital depreciation to be charged as an operating expense each year until the total capital depreciation has been charged.

The amount and rate at which equipment may be depreciated are set by the federal government (Internal Revenue Service of the U.S. Treasury Department). The regulations that cover the capital depreciation change often. Both the current method of depreciation suggested by the IRS and several of the techniques that have been used in the past to depreciate capital investment are presented. Example 9.20 illustrates the need for depreciation of capital.

Example 9.20

Consider a person who owns a business with the following annual revenue and expenses:

Revenue from sales	$ 356,000
Rent	($ 22,000)
Employee salaries	($ 100,000)
Employee benefits	($ 32,000)
Utilities	($ 7000)
Miscellaneous expenses	($ 5000)
Overhead expenses	($ 40,000)
Before-tax profit	$ 150,000

The owner of the business decides that, in order to improve the manufacturing operation, a new packing and labeling machine must be bought for $100,000, which has a useful operating life of four years and can be sold for $2000 scrap value at that time. This is estimated to increase sales by 5% per year. The only additional cost is an extra $1000/y in utilities. The new, before-tax profit is estimated to be

Before-Tax Profit = $150,000 + 17,800 – 1000 = $166,800/y, or an increase of $16,800/y

Using a before-tax basis, it can be seen that the $100,000 investment yields $16,800/y. The alternative to buying the new machine is to invest money in a mutual fund that yields 10% p.a., before tax. At face value, the investment in the new machine looks like a winner. However, a close look at the cash flows for each case is suggested (Table E9.20).

Table E9.20 Cash Flows for Both Investment Opportunities

Year	Cash Flow for Investment in Machine (All $ Figures in 1000)	Cash Flow for Investment in Mutual Fund (All $ Figures in 1000)
0	−100	−100
1	16.8	10
2	16.8	11
3	16.8	12.1
4	16.8 + 2.0	13.31 + 100
Total	**−30.8**	**46.41**

Although the yearly return for buying the machine looks much better than that for the investment in the mutual fund, the big difference is that at the end of the fourth year the owner can recover the initial investment from the mutual fund, but the machine is worth only $2000. From this example, it can be seen that the $100,000 - $2000 = $98,000 investment in the machine is really a long-term expense, and the owner should be able to deduct it as a legitimate operating expense. Depreciation is the method that the government allows for businesses to obtain operating expense credits for capital investments.

9.7.1 Fixed Capital, Working Capital, and Land

When the depreciation of capital investment is discussed, care must be taken to distinguish between what can and cannot be depreciated. In general, the total capital investment in a chemical process is made up of two components:

$$\text{Total Capital Investment} = \text{Fixed Capital} + \text{Working Capital} \tag{9.21}$$

Fixed capital is all the costs associated with building the plant and was covered in Chapter 7 (either total module cost or grassroots cost). The only part of the fixed capital investment that cannot be depreciated is the **land**, which usually represents only a small fraction of the total.

Working capital is the amount of capital required to start up the plant and finance the first few months of operation before revenues from the process start. Typically, this money is used to cover salaries, raw material inventories, and any contingencies. The working capital will be recovered at the end of the project and represents a float of money to get the project started. This concept is similar to that of paying the first and last month's rent on an apartment. The last month's rent is fully recoverable at the end of the lease but must be paid at the beginning. Because the working capital is fully recoverable, it cannot be depreciated. There are different methods for estimating working capital. One might be a fraction (15%–20%) of the fixed capital investment. Another might be four to six months of raw materials and utility costs.

9.7.2 Different Types of Depreciation

First the terms that are used to evaluate depreciation are introduced and defined.

Fixed Capital Investment, FCI_L**:** This represents the fixed capital investment to build the plant minus the cost of land and represents the depreciable capital investment.

Salvage Value, S**:** This represents the fixed capital investment of the plant, minus the value of the land, evaluated at the end of the plant life. Usually, the equipment salvage (scrap) value represents a small fraction of the initial fixed capital investment. Often the salvage value of the equipment is assumed to be zero.

Life of the Equipment, n**:** This is specified by the U.S. Internal Revenue Service (IRS). It does not reflect the actual working life of the equipment but rather the time allowed by the IRS for equipment depreciation. Chemical process equipment currently has a depreciation class life of 9.5 years [1].

Total Capital for Depreciation: The total amount of depreciation allowed is the difference between the fixed capital investment and the salvage value.

$$D = FCI_L - S$$

Yearly Depreciation: The amount of depreciation varies from year to year. The amount allowed in the kth year is denoted d_k.

Book Value: This is the amount of the depreciable capital that has not yet been depreciated.

$$BV_k = FCI_L - \sum_{1}^{k} d_j$$

A discussion of three representative depreciation methods that have been widely used to determine the depreciation allowed each year is provided. Currently, only the straight-line and double declining balance methods are approved by the IRS, though the actual depreciation schedules suggested by the IRS are a combination of these two methods. The sum-of-the-years-digits method has been used previously and is included here for completeness.

Straight-Line Depreciation Method, ***SL*:** An equal amount of depreciation is charged each year over the depreciation period allowed. This is shown as

$$d_k^{SL} = \frac{[FCI_L - S]}{n} \tag{9.22}$$

Sum of the Years Digits Depreciation Method, ***SOYD*:** The formula for calculating the depreciation allowance is as follows:

$$d_k^{SOYD} = \frac{[n + 1 - k][FCI_L - S]}{\frac{n}{2}[n + 1]} \tag{9.23}$$

The method gets its name from the denominator of Equation (9.23), which is equal to the sum of the number of years over which the depreciation is allowed:

$$1 + 2 + 3 \ldots + n = (n)(n + 1)/2$$

For example, if $n = 7$, then the denominator equals 28.

Double Declining Balance Depreciation Method, ***DDB*:** The formula for calculating the depreciation allowance is as follows:

$$d_k^{DDB} = \frac{2}{n}\left[FCI_L - \sum_{j=0}^{j=k-1} d_j\right] \tag{9.24}$$

In the declining balance method, the amount of depreciation each year is a constant fraction of the book value, BV_{k-1}. The word *double* in *DDB* refers to the factor 2 in Equation (9.24). Values other than 2 are sometimes used; for example, for the 150% declining balance method, 1.5 is substituted for the 2 in Equation (9.24).

In this method, the salvage value does not enter into the calculations. It is not possible, however, to depreciate more than the value of *D*. To avoid this problem, the final year's depreciation is reduced to obtain this limiting value.

Example 9.21 illustrates the use of each of the above formulas to calculate the yearly depreciation allowances.

Example 9.21

The fixed capital investment (excluding the cost of land) of a new project is estimated to be $150.0 million, and the salvage value of the plant is $10.0 million. Assuming a seven-year equipment life, estimate the yearly depreciation allowances using the following:

1. The straight-line method
2. The sum of the years digits method
3. The double declining balance method

Solution

It is given that FCI_L = $150 × 10^6, S = $10.0 × 10^6, and n = 7 years.

Sample calculations for year 2 give the following:

For straight-line depreciation, using Equation (9.22),

$$d_2 = (\$150 \times 10^6 - \$10 \times 10^6)/7 = \$20 \times 10^6$$

For SOYD depreciation, using Equation (9.23),

d_2 = (7 + 1 – 2)($150 × 10^6 – $10 × 10^6)/28 = $30 × 106

For double declining balance depreciation, using Equation (9.24),

d_2 = (2/7)($150 × 10^6 – $42.86 × 10^6) = $30.6 × 10^6

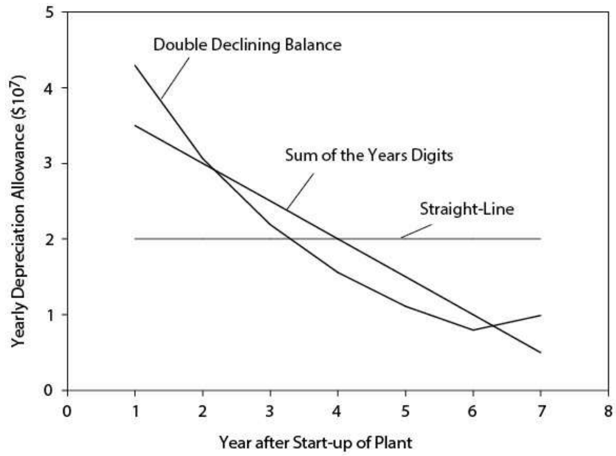
A summary of all the calculations is given in Table E9.21 and presented graphically in Figure E9.21.

Table E9.21 Calculations and Results for Example 9.21: The Depreciation of Capital Investment for a New Chemical Plant (All Values in $$10^7$)

Year (k)	d_k^{SL}	d_k^{SOYD}	d_k^{DDB}	Book Value $FCI_L - Sd_k^{DDB}$
0				(15 – 0) = 15
1	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-1)(15-1)}{28^*} = 3.5$	$\frac{(2)(15)}{7} = 4.29$	(15 – 4.29) = 10.71
2	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-2)(15-1)}{28^*} = 3.0$	$\frac{(2)(10.71)}{7} = 3.06$	(10.71 – 3.06) = 7.65
3	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-3)(15-1)}{28^*} = 2.5$	$\frac{(2)(7.65)}{7} = 2.19$	(7.65 – 2.19) = 5.46
4	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-4)(15-1)}{28^*} = 2.0$	$\frac{(2)(5.46)}{7} = 1.56$	(5.46 – 1.56) = 3.90
5	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-5)(15-1)}{28^*} = 1.5$	$\frac{(2)(3.90)}{7} = 1.11$	(3.90 – 1.11) = 2.79
6	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-6)(15-1)}{28^*} = 1.0$	$\frac{(2)(2.79)}{7} = 0.80$	(2.79 – 0.80) = 1.99
7	$\frac{(15-1)}{7} = 2$	$\frac{(7+1-7)(15-1)}{28^*} = 0.5$	1.99 – 1.0 = 0.99†	(1.99 – 0.99) = 1.00
Total	14.0	14.0	14.0	1.0 = Salvage Value†

*Sum of digits: $[n + 1]n/2 = [7 + 1]\ 7/2 = 28$

†The depreciation allowance in the final year of the double declining balance method is adjusted to give a final book value equal to the salvage value.



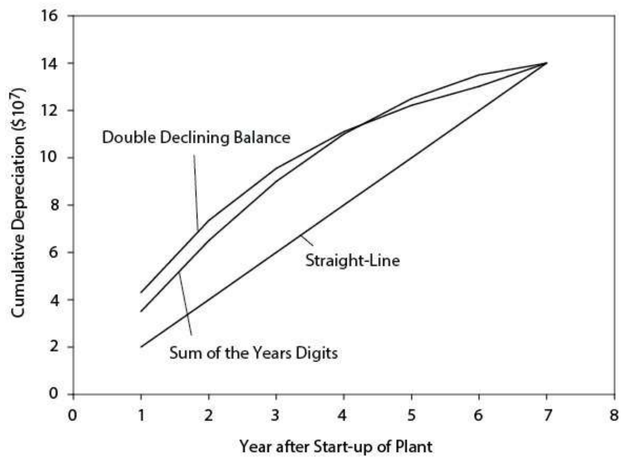


Figure E9.21 Yearly Depreciation Allowances and Cumulative Depreciation Amounts for Example 9.21

From Figure E9.21 it is seen as follows:

1. The depreciation values obtained from the sum of the years digits are similar to those obtained from the double declining balance method.
2. The double declining balance method has the largest depreciation in the early years.
3. The straight-line method represents the slowest depreciation in the early years.

The SOYD and the DDB methods are examples of accelerated depreciation schemes (relative to the straight line). It is shown in Section 9.8 that accelerated depreciation has significant economic advantages over the straight-line method.

> Capital investment can be depreciated only in accordance with current tax regulations.

9.7.3 Current Depreciation Method (2017): Modified Accelerated Cost Recovery System (MACRS)

The current federal tax law is based on MACRS, using a half-year convention. All equipment is assigned a class life, which is the period over which the depreciable portion of the investment may be discounted. Most equipment in a chemical plant has a class life of 9.5 years [1] with no salvage value. This means that the capital investment may be depreciated using a straight-line method over 9.5 years. Alternatively, a MACRS method over a shorter period of time may be used, which is five years for this class life. In general, it is better to depreciate an investment as soon as possible. This is because the more the depreciation is in a given year, the less taxes paid. As shown earlier in this chapter, "money now is worth more than the same amount in the future"; therefore, it is better to pay less in taxes at the beginning of a project than at the end.

The MACRS method uses a double declining balance method and switches to a straight-line method when the straight-line method yields a greater depreciation allowance for that year. The straight-line method is applied to the remaining depreciable capital over the remaining time allowed for depreciation. The half-year convention assumes that the equipment is bought midway through the first year for which depreciation is allowed. In the first year, the depreciation is only half of that for a full year. Likewise in the sixth (and last) year, the depreciation is again for one-half year. The depreciation schedule for equipment with a 9.5-year class life and 5-year recovery period, using the MACRS method, is shown in Table 9.2.

Table 9.2 Depreciation Schedule for MACRS Method for Equipment with a 9.5-Year Class Life and a 5-Year Recovery Period [1]

Year	Depreciation Allowance (% of Capital Investment)
1	20.00
2	32.00
3	19.20
4	11.52
5	11.52
6	5.76

Example 9.22 illustrates the method by which the MACRS depreciation allowances in Table 9.2 are obtained.

Example 9.22

Show how Table 9.2 is obtained.

Solution

The basic approach is to use the double declining balance (DDB) method and compare the result with the straight-line (SL) method for the remaining depreciable capital over the remaining period of time. The MACRS method requires depreciation of the total FCI_L, without regard for the salvage value. Calculations are given below, using a basis of $100:

$$\text{For DDB, } d_k^{DDB} = \frac{2}{5}\left[100 - \sum_{j=0}^{k-1} d_j^{DDB}\right]$$

$$\text{For SL, } d_k^{SL} = \frac{\text{undepreciated capital}}{\text{remaining time for depreciation}}$$

k	d_k^{DDB} (← ½ year convention)		d_k^{SL} time remaining
1	0.4(100)(0.5) = **20**		
2	0.4(100 − 20) = **32**		(100 − 20)/4.5 = 17.78
3	0.4(100 − 52) = **19.2**		(100 − 52)/3.5 = 13.71
4	0.4(100 − 71.2) = **11.52**	⟶	(100 − 71.2)/2.5 = **11.52**
5	0.4(100 − 82.72) = 6.91		(100 − 82.72)/1.5 = **11.52**
6		½ year convention ⟶	(0.5)(100 − 94.24)/0.5 = **5.76**

9.8 TAXATION, CASH FLOW, AND PROFIT

Taxation has a direct impact on the profits realized from building and operating a plant. Tax regulations are complex, and companies have tax accountants and attorneys to ensure compliance and to maximize the benefit from these laws. When individual projects are considered or similar projects are compared, accounting for the effect of taxes is required. Taxation rates for companies and the laws governing taxation change frequently. The current tax rate schedule (as of 2016) is given in Table 9.3.

Table 9.3 Federal Tax Rate Schedule for Corporations [2]

Range of Net Taxable Income	Taxation Rate
> $0 and ≤ $50,000	15%
> $50,000 and ≤ $75,000	$7500 + 25% of amount over $50,000
> $75,000 and ≤ $100,000	$13,750 + 34% of amount over $75,000
> $100,000 and ≤ $335,000	$22,250 + 39% of amount over $100,000
> $335,000 and ≤ $10 million	$113,900 + 34% of amount over $335,000
> $10 million and ≤ $15 million	$3,4000,000 + 35% of amount over $10 million
> $15 million and ≤ $18.333 million	$5,150,000 + 38% of amount over $15 million
> $18.333 million	35%

For most large corporations, the basic federal taxation rate is 35%. In addition, corporations must also pay state, city, and other local taxes. The overall taxation rate is often in the range of 40% to 50%. The taxation rate used in the problems at the back of this chapter will vary and may be as low as 30%. For the economic analysis of a proposed (current) process, clearly it is important to use the correct taxation rate, which will, in turn, depend on the location of the proposed process.

Table 9.4 provides the definition of important terms and equations used to evaluate the cash flow and the profits produced from a project.

Table 9.4 Evaluation of Cash Flows* and Profits* in Terms of Revenue (*R*), Cost of Manufacturing (*COM*), Depreciation (*d*), and Tax Rate (*t*)

	Description	Formula	Equation
Expenses	= Manufacturing Costs + Depreciation	$= COM_d + d$	(9.25)
Income Tax	= (Revenue - Expenses) (Tax Rate)	$= (R - COM_d - d)(t)$	(9.26)
After-Tax	= Revenue - Expenses - Income Tax	$= (R - COM_d - d)(1 - t)$	(9.27)
(Net) Profit			
After-Tax	= Net Profit + Depreciation	$= (R - COM_d - d)(1 - t) + d$	(9.28)
Cash Flow			
Variables:			
t	Tax Rate		Constant
COM_d	Cost of Manufacture Excluding		(8.2)
	Depreciation		
d	Depreciation: Depends upon Method		(9.22)
	Used		(9.23)
R	Revenue from Sales		(9.24)
*To obtain before-tax values, set the tax rate (t) to zero.			

The equations from Table 9.4 are used in Example 9.23.

Example 9.23

For the project given in Example 9.21, the manufacturing costs, excluding depreciation, are \$30 million/y, and the revenues from sales are \$75 million/y. Given the depreciation values calculated in Example 9.21, calculate the following for a 10-year period after start-up of the plant:

1. The after-tax profit (net profit)
2. The after-tax cash flow, assuming a taxation rate of 30%

Solution

From Equations (9.27) and (9.28) (all numbers are in $\$10^6$),

After-tax profit $= (75 - 30 - d_k)(1 - 0.3) = 31.5 - (0.7)(d_k)$

After-tax cash flow $= (75 - 30 - d_k)(1 - 0.3) + d_k = 31.5 + (0.3)(d_k)$

A sample calculation for year 1 ($k = 1$) is provided:

From Example 9.21, $d_1^{SL} = 20$, $d_1^{SOYD} = 35$, and $d_1^{DDB} = 42.9$.

	SL	SOYD	DDB
Profit after Tax	17.5	7.0	1.47
Cash Flow after Tax	37.5	42.0	44.37

The calculations for years 1 through 10 are plotted in Figure E9.23. From this plot, it can be seen that the cash flow at the start of the project is greatest for the DDB method and lowest for the SL method.

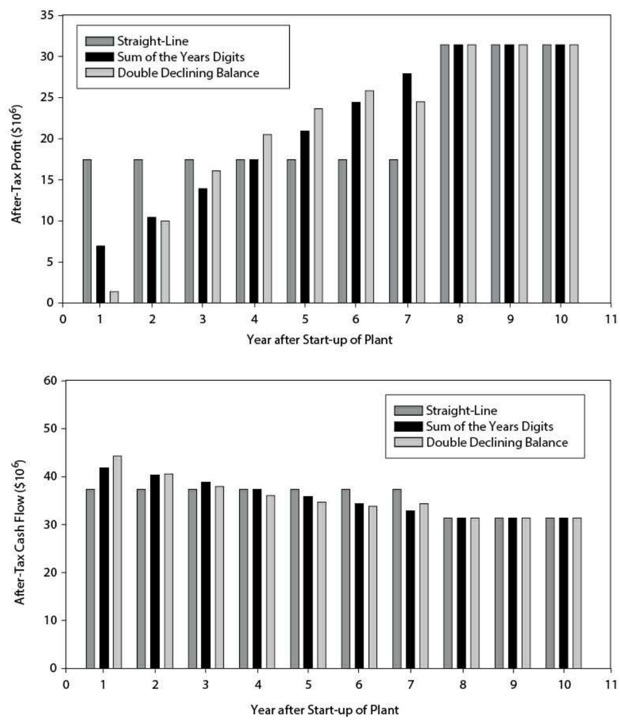


Figure E9.23 Comparison of the After-Tax Profit and Cash Flow Using Different Depreciation Schedules from Example 9.23

The sum of the profits and cash flows over the 10-year period are $217 million and $357 million, respectively. These totals are the same for each of the depreciation schedules used. The difference between the cash flows and the profits is seen to be the depreciation ($357 – 217 = $140 million).

Example 9.23 demonstrated how different depreciation schedules affect the after-tax cash flow. The accelerated schedules for depreciation provided the greatest cash flows in the early years. Because money earned in early years has a greater value than money earned in later years, the accelerated schedule of depreciation is the most desirable alternative.

9.9 SUMMARY

In this chapter, the basics of economic analysis required to evaluate project profitability were covered. The material presented in this chapter is founded on the principle that

Money + Time = More Money

To benefit from this principle, it is necessary to have resources to make an investment and the time to allow the investment to grow. When this principle is applied to chemical processes, the revenue or additional money is generated when low-value materials and services are converted into high-value materials and services.

A central concept identified as the time value of money grows out of this principle. This principle is applied to a wide range of applications, from personal financial management to the analysis of new chemical plants.

The use of cash flow diagrams to visualize the timing of cash flows and to manage cash flows during a project was illustrated. A shorthand notation for the many discount factors involved in economic calculations (factors that account for the time value of money) were introduced to simplify cash flow calculations. Other items necessary for a comprehensive economic evaluation of a chemical plant were covered and included depreciation, taxation, and the evaluation of profit and cash flow.

Applications involving these principles and concepts directly relating to chemical plants will be pursued in Chapter 10.

WHAT YOU SHOULD HAVE LEARNED

- That money today is worth more than money tomorrow
- Simple versus compound interest
- How to calculate the time value of money involving present value, future value, and annuity
- How to represent calculations of the time value of money on a cash flow diagram
- How to include taxation and depreciation
- The significance and impact of inflation

REFERENCES

1. *How to Depreciate Property*, Publication 946, Department of the Treasury, Internal Revenue Service, February 2017, available at https://www.irs.gov/pub/irs-pdf/p946.pdf.
2. *Corporations, Property*, Publication 542, Department of the Treasury, Internal Revenue Service, December 2016, available at https://www.irs.gov/pub/irs-pdf/p542.pdf.

SHORT ANSWER QUESTIONS

1. What is the difference between simple and compound interest? Provide an example.
2. What is the difference between the nominal annual interest

rate and the effective annual interest rate? When are these two rates equal?

3. You work in the summer and receive three large, equal monthly paychecks in June, July, and August. You are living at home, so there are no living expenses. You use this money to pay your tuition for the next academic year in September and in January. You also use this money to pay monthly living expenses during the academic year, which are assumed to be cash flows once per month from September through May. Illustrate these cash flows on a cash flow diagram.
4. Define the term *annuity*.
5. The value of the Chemical Engineering Plant Cost Index (CEPCI) at the beginning of 2016 is 557. If at the same time next year the value has risen to 594, what will be the average inflation rate for the year?
6. Discuss the differences and similarities between interest, inflation, and the time value of money.
7. What term describes the method by which a fixed capital investment can be used to reduce the tax that a company pays?
8. What is depreciation? Explain how it affects the economic analysis of a new chemical process.
9. Why is it advantageous to use an accelerated depreciation schedule?
10. "In general, it is better to depreciate the fixed capital investment as soon as allowable." Give one example of when this statement would not be true.
11. What is the difference between after-tax profit and after-tax cash flow? When are these two quantities the same?

PROBLEMS

12. You need to borrow $1000 for an emergency. You have two alternatives. One is to borrow from a bank. The other is to borrow from your childhood friend Greta Ganav, who is in the "private" financing business. Because you and Greta go way back, she will give you her preferred rate, which is $5/week until you pay back the entire loan.

    1. What is Greta's effective annual interest rate?
    2. Your alternative is to borrow from a bank, where the interest is compounded monthly. What nominal interest rate would make you choose the bank over Greta?
    3. If the bank's interest rate is 7% p.a., compounded monthly, how many months would it be until the interest paid to Greta equaled the interest paid to the bank?

13. Consider the following three investment schemes:

9.5% p.a. (nominal rate) compounded daily

10.0% p.a. (nominal rate) compounded monthly

10.5% p.a. (nominal rate) compounded quarterly

1. Which investment scheme is the most profitable, assuming that the initial investment is the same for each case?
2. What is the effective annual interest rate of the best scheme?
3. What is the nominal interest rate of the best scheme when compounded continuously?

14. At an investment seminar that I recently attended, I learned about something called "the Rule of 72." According to the person in charge of the seminar, a good estimate for finding how long it takes for an investment to double is given by the following equation:

$$\text{Number of years to double investment} = \frac{72}{\text{effective annual interestrate(in\%)}}$$

Using what you know about the time value of money, calculate the error in using the above equation to estimate how long it takes to double an investment made at the following effective annual interest rates: 5%, 7.5%, and 10%. Express your answers as % error to two significant figures. Comment on the Rule of 72 and its accuracy for typical financial calculations today.

15. You invested $5000 eight years ago, and you want to determine the value of the investment now, at the end of year 8. During the past eight years, the nominal interest rate has fluctuated as follows:

Year	Nominal Interest Rate (% p.a.)
1	4
2	5
3	7
4	8
5	6
6	4
7	5
8	4

If your investment is compounded daily, how much is it worth today? (Ignore the effect of leap years.)

16. What are the differences between investing $15,000 at 9% p.a. for 15 years when compounded yearly, quarterly, monthly, daily, and continuously? Ignore the effect of leap years.

17. One bank advertises an interest rate on a certificate of deposit (CD) to be 4% compounded daily. Another bank

advertises a CD with a 4.1% effective annual rate. In which CD would you invest?

18. You take out a new-car loan from an internet bank that compounds interest weekly and requires weekly payments. The interest rate is 5.5%. What is the weekly payment for a $20,000, five-year loan, assuming one year is exactly 52 weeks?

19. You want to begin an investment plan to save for your daughter's college education. Because you believe that your newly born daughter will be a genius (just like you!), you are assuming the cost of an Ivy League education. You plan to put money into an investment account, at the end of each year, for the next 18 years. You believe that you will need about $75,000/y, each year, 19 through 22 years from now.

    1. Draw a discrete cash flow diagram for this situation.
    2. How much would you have to invest each year to pay for college and have a zero balance at the end of year 22? The effective annual interest rate of your investment is 8%.
    3. What interest rate would be needed if you could invest only $5000/y?

20. You begin an investment plan by putting $10,000 in an account that you assume will earn 8% annually. For the next 25 years, you add $5000/y. In anticipation of buying a house with a 15-year mortgage, you expect to need a one-time down payment of $55,000 at the end of year 8. You anticipate being able to make the monthly mortgage payment without affecting the yearly contribution to the savings plan.

    1. Draw a discrete, nondiscounted cash flow diagram for this situation.
    2. Will you have the down payment at the end of year 8?
    3. What will be the value of your savings account at the end of year 25?

21. You begin work on June 1 and work until August 31, and receive pay on the last day of the month. Your expenses for these three months are $1500/mo. At the end of September, you make a $7000 payment, and you make an identical payment at the end of January. Your expenses from September 1 through May 31 are $1000/mo.

    1. Draw a discrete, nondiscounted cash flow diagram for this situation.
    2. If you earn 4% interest, compounded monthly, on the money until it is spent, what monthly salary is required from June 1 through August 31 to break even on May 31?
    3. If you earn 4% interest, compounded monthly, on the money until it is spent and the salary is $4000/mo, how much would you have to earn each month from a different job from September through May to break even on May 31?
    4. What situation is depicted in this problem?

22. The cash flows for a bank account are described by the discrete CFD in Figure P9.22. The bank account has an

effective annual interest rate of 4.5%.

1. Calculate the future value of all cash flows after 15 years.
2. Calculate the future value of all cash flows after 25 years assuming that there are no more transactions after year 15.

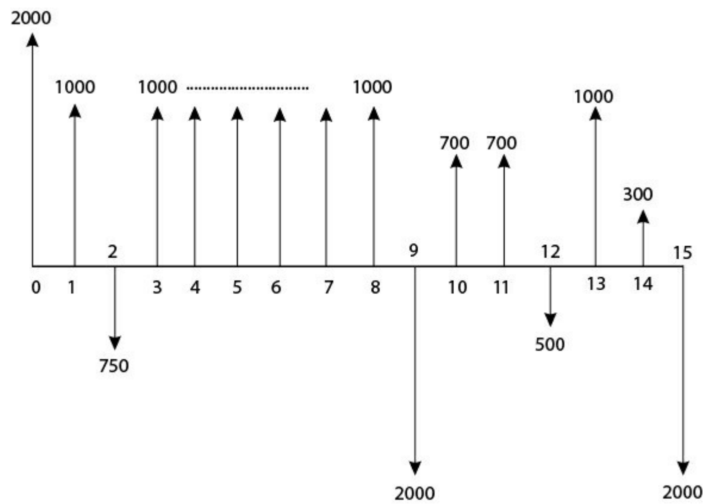


Figure P9.22 Cash Flow Diagram for Problem 9.22

23. In a tax-deferred investment plan such as a 401k, you can invest money deducted from your gross salary before taxes are withheld. Most companies provide some sort of matching contribution. Suppose you invest $10,000/y via salary deduction, with an equal company match, for 40 years. What is the future value of this investment after 40 years? Repeat this calculation for the cases where you delay the investment plan for 10, 20, and 30 years. Assume an 8% annual rate of return.

24. You begin to contribute to an investment plan with your company immediately after graduation, when you are 23 years old. Your contribution plus your company's contribution totals $6000/y. Assume that you work for the same company for 40 years.

1. What effective annual interest rate is required for you to have $1 million in 40 years?
2. Repeat Part (a) for $2 million.
3. What is the future value of this investment after 40 years if the effective annual interest rate is 7% p.a.?

25. A 401k (and similar plans) is an investment vehicle available to most individuals to save for retirement. The benefit is that the investment is made before taxes (by payroll deduction), and interest is not taxed until the money is withdrawn. In theory, most retirees are in a lower tax bracket than when they worked. Under current laws (2016), the maximum annual contribution to all such plans in 2016 is $18,000 until age 50 and $24,000 thereafter, including the year one turns 50. Suppose that the maximum contribution is made every year for 40 years, beginning at age 25 until age 65.

(*Note*: Employers may match this contribution, but this is not considered here.)

1. What is the future value of this retirement fund after 40 years, assuming an effective annual interest rate of 9%?
2. What effective annual interest rate is required for the future value of this investment to be $2 million after 40 years?

26. You plan to finance a new car by borrowing $25,000. The interest rate is 7% p.a., compounded monthly. What is your monthly payment for a three-year loan, a four-year loan, and a five-year loan?
27. You have just purchased a new car by borrowing $20,000 for four years. Your monthly payment is $500. What is your nominal interest rate if compounded monthly?
28. You plan to borrow $200,000 to purchase a new house. The nominal interest rate is fixed at 6.5% p.a., compounded monthly.

1. What is the monthly payment on a 30-year mortgage?
2. What is the monthly payment on a 15-year mortgage?
3. What is the difference in the amount of interest paid over the lifetime of each loan?

29. You just borrowed $225,000 to purchase a new house. The monthly payment (before taxes and insurance) is $1791 for the 25-year loan. What is the effective annual interest rate?
30. You just borrowed $250,000 to purchase a new house. The nominal interest rate is 6% p.a., compounded monthly, and the monthly payment is $1612 for the loan. What is the duration of the loan?
31. A home-equity loan involves borrowing against the equity in a house. For example, if your house is valued at $250,000 and you have been paying the mortgage for a sufficient amount of time, you may owe only $150,000 on the mortgage. Therefore, your **equity** in the home is $100,000. You can use the home as collateral for a loan, possibly up to $100,000, depending on the bank's policy. Home-equity loans often have shorter durations than regular mortgages. Suppose that you take a home-equity loan of $50,000 for the down payment on a new house in a new location because you have a new job. When you complete the sale of your old house, the home-equity loan will be paid off at the closing. The home-equity loan terms are a 10-year term at 7% p.a., compounded monthly, but you must also pay 0.05% of the original home-equity loan principal each month. What is the monthly payment?
32. Your company is trying to determine whether to spend $500,000 in process improvements. The projected cash flow increases based on the process improvements are as follows:

Year	Annual Increased Cash Flow ($thousands)

1	25
2	75
3	100
4	125
5	250

The alternative is to do nothing and leave the $500,000 in the investment portfolio earning interest. What interest rate is required in the investment portfolio for the better choice to be to do nothing?

33. It is necessary to evaluate the profitability of proposed improvements to a process prior to obtaining approval to implement changes. For one such process, the capital investment (end of year 0) for the project is $250,000. There is no salvage value. In years 1 and 2, you expect to generate an after-tax cash flow from the project of $60,000/y. In years 3–8, you expect to generate an after-tax cash flow of $50,000/y. Assume that the investments and cash flows are single transactions occurring at the end of the year. Assume an effective annual interest rate of 9%.

    1. Draw a discrete cash flow diagram for this project.
    2. Draw a cumulative, discounted (to year 0) cash flow diagram for this project.
    3. What is the future value of this project at the end of year 8?
    4. Instead of investing in this project, the $250,000 could remain in the company's portfolio. What rate of return on the portfolio is needed to equal the future value of this project at the end of year 8? Would you invest in the project, or leave the money in the portfolio?

34. In Problem 9.33, the after-tax cash flow figures were generated using a taxation rate of 45% and a straight-line depreciation over the eight-year project. Calculate the yearly after-tax cash flows if the five-year MACRS depreciation schedule were used.

35. You are evaluating the profitability potential of a process and have the following information. The criterion for profitability is a 15% rate of return over ten operating years. The equipment has zero salvage value at the end of the project.

    Fixed capital investment (including land) in four installments (all values are in millions of dollars as one transaction at the end of the year):

Year 0 land	$10
Year 1 FCI installment 1	$20
Year 2 FCI installment 2	$30
Year 3 FCI installment 3	$20
Start-up capital at end of year 3	$10
Positive cash flow years 4–13	$25

1. Draw a discrete, discounted cash flow diagram for this process.
2. Draw a cumulative, discounted cash flow diagram for this process.
3. What is the present value (at end of year 0) for this process?
4. What is the future value at the end of year 13?
5. What would the effective annual interest rate have to be so that the present value (end of year 0) of this investment is zero? (This interest rate is known as the DCFROR, and it will be discussed in the next chapter.)
6. What is your recommendation regarding this process? Explain.

36. What are the MACRS depreciation allowances for recovery periods of four, six, and nine years?

37. For a new process, the land was purchased for $10 million. The fixed capital investment, paid at the end of year 0, is $165 million. The working capital is $15 million, and the salvage value is $15 million. The estimated revenue from years 1 through 10 is $70 million/y, and the estimated cost of manufacture over the same time period is $25 million/y. The internal hurdle rate (interest rate) is 14% p.a., before taxes, and the taxation rate is 40%.

1. Draw a discrete, nondiscounted (before-tax) cash flow diagram for this process.
2. Determine the yearly depreciation schedule using the five-year MACRS method.
3. Determine the after-tax profit for each year.
4. Determine the after-tax cash flow for each year.
5. Draw a discrete, discounted (to year 0) cash flow diagram for this process.
6. Draw a cumulative, discounted (to year 0) cash flow diagram for this process.
7. What is the present value (year 0) of this process?

Chapter 10: Profitability Analysis

WHAT YOU WILL LEARN

- There are different methods for estimating the profitability of a proposed chemical process.
- These methods do not always give the same result.
- Certain methods are more appropriate for specific situations.
- The impact of uncertainty can be included in these profitability estimates using a Monte-Carlo analysis.

This chapter will explain how to apply the techniques of economic analysis developed in Chapter 9. These techniques will be used to assess the profitability of projects involving both capital expenditures and yearly operating costs. A variety of projects will be examined, ranging from large multimillion-dollar ventures to much smaller process improvement projects. Several criteria for profitability will be discussed and applied to the evaluation of process and equipment alternatives. The first concept is the profitability criteria for new large projects.

10.1 A TYPICAL CASH FLOW DIAGRAM FOR A NEW PROJECT

A typical cumulative, after-tax cash-flow diagram (CFD) for a new project is illustrated in Figure 10.1. It is convenient to relate profitability criteria to the cumulative CFD rather than the discrete CFD. The timing of the different cash flows is explained below.

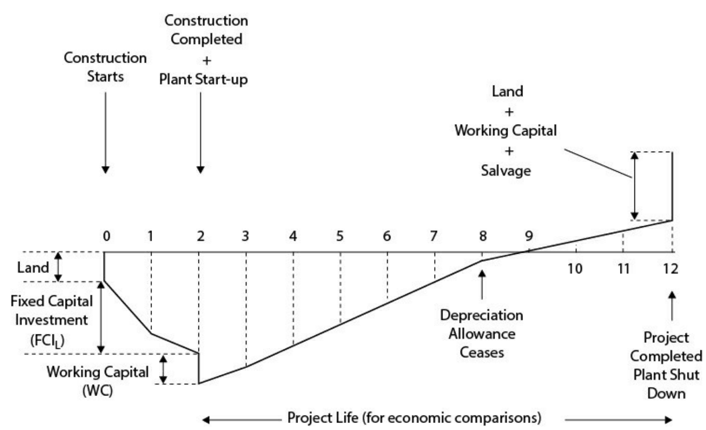


Figure 10.1 A Typical Cumulative Cash Flow Diagram for the Evaluation of a New Project

In the economic analysis of the project, it is assumed that any new land purchases required are done at the start of the project, that is, at time zero. After the decision has been made to build a new chemical plant or expand an existing facility, the construction phase of the project starts. Depending on the size and scope of the project, this construction may take anywhere from six months to three years to complete. In the example shown in Figure 10.1, a typical value of two years for the time from project initiation to the startup of the plant has been assumed. Over the two-year construction phase, there is a major capital outlay. This represents the fixed capital expenditures for purchasing and installing the equipment and auxiliary facilities required to run the plant (see Chapter 7). The distribution of this fixed capital investment is usually slightly larger toward the beginning of construction, and this is reflected in Figure 10.1. At the end of the second year, construction is finished and the plant is started. At this point, the additional expenditure for working capital required to float the first few months of operations is shown. This is a one-time expense at the startup of the plant and will be recovered at the end of the project.

After startup, the process begins to generate finished products for sale, and the yearly cash flows become positive. This is reflected in the positive slope of the cumulative CFD in Figure 10.1. Usually the revenue for the first year after startup is less than in subsequent years due to "teething" problems in the plant; this is also reflected in Figure 10.1. The cash flows for the early years of operation are larger than those for later years due to the effect of the depreciation allowance discussed in Chapter 9. The time used for depreciation in Figure 10.1 is six years. The time over which the depreciation is allowed depends on the IRS regulations and the method of depreciation used.

In order to evaluate the profitability of a project, a life for the process must be assumed. This is not usually the working life of the equipment, nor is it the time over which depreciation is allowed. It is a specific length of time over which the profitability of different projects is to be compared. Lives of 10, 12, and 15 years are commonly used for this purpose. It is necessary to standardize the project life when comparing different projects. This is because profitability is directly related to project life, and comparing projects using different lives biases the results.

Usually, chemical processes have anticipated operating lives much greater than ten years. If much of the equipment in a specific process is not expected to last for a ten-year period, then the operating costs for that project should be adjusted. These operating costs should reflect a much higher maintenance cost to include the periodic replacement of equipment necessary for the process to operate the full ten years. A project life of ten years will be used for the examples in the next section.

From Figure 10.1, a steadily rising cumulative cash flow is observed over the ten operating years of the process, that is, years 2 through 12. At the end of the ten years of operation, that is, at the end of year 12, it is assumed that the plant is closed down and that all the equipment is sold for its salvage or scrap value, that the land is also sold, and that the working capital is recovered. This additional cash flow, received on closing down the plant, is shown by the upward-pointing vertical line in year 12. It must be remembered that in reality, the plant will most likely *not* be closed down; it is only assumed that it will be to perform the economic analysis.

The question that must now be addressed is how to evaluate the profitability of a new project. Looking at Figure 10.1, it can be seen that at the end of the project the cumulative CFD is positive. Does this mean that the project will be profitable? The answer to this question depends on whether the value of the income earned during the time the plant operated is smaller or greater than the investment made at the beginning of the project. Therefore, the time value of money must be considered when evaluating profitability. The following sections examine different ways to evaluate project profitability.

10.2 PROFITABILITY CRITERIA FOR PROJECT EVALUATION

There are three bases used for the evaluation of profitability:

1. Time
2. Cash
3. Interest rate

For each of these bases, discounted or nondiscounted techniques may be considered. The nondiscounted techniques do not take into account the time value of money and are *not* recommended for evaluating new, large projects. Traditionally, however, such methods have been and are still used to evaluate smaller projects, such as process improvement schemes. Examples of both types of methods are presented for all three bases.

10.2.1 Nondiscounted Profitability Criteria

Four nondiscounted profitability criteria and the graphical interpretation of these profitability criteria are illustrated in Figure 10.2. Each of the four criteria is explained below.

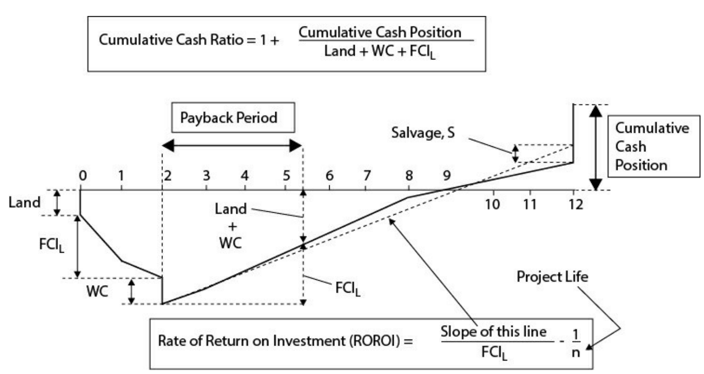


Figure 10.2 Illustration of Nondiscounted Profitability Criteria

Time Criterion. The term used for this criterion is the **payback period (PBP),** also known by a variety of other names, such as payout period, payoff period, and cash recovery period. The payback period is defined as follows: *PBP* = Time required, after startup, to recover the fixed capital investment, FCI_L, for the project

The payback period is shown as a length of time on Figure 10.2. Clearly, the shorter the payback period, the more profitable the project.

Cash Criterion. The criterion used here is the **cumulative cash position (CCP),** which is simply the worth of the project at the end of its life. For criteria using cash or monetary value, it is difficult to compare projects with dissimilar fixed capital investments, and sometimes it is more useful to use the **cumulative cash ratio (CCR),** which is defined as

$$CCR = \frac{\text{Sum of All Positive Cash Flows}}{\text{Sum of All Negative Cash Flows}}$$

The definition effectively gives the cumulative cash position normalized by the initial investment. Projects with cumulative cash ratios greater than one are potentially profitable, whereas those with ratios less than unity cannot be profitable.

Interest Rate Criterion. The criterion used here is called the **rate of return on investment (ROROI)** and represents the nondiscounted rate at which money is made from a fixed capital investment. The definition is given as

$$ROROI = \frac{\text{Average Annual Net Profit}}{\text{Fixed Capital Investment}(FCI_L)}$$

The annual net profit in this definition is an average over the life of the plant after startup.

The use of fixed capital investment, FCI_L, in the calculations for payback period and rate of return on investment given above seems reasonable, because this is the capital that must be recovered by project revenue. Many alternative definitions for these terms can be found, and sometimes the total capital investment (FCI_L + *WC* + Land) is used instead of fixed capital investment. When the plant has a salvage value (*S*), the fixed capital investment minus the salvage value ($FCI_L - S$) could be used instead of FCI_L. However, because the salvage value is usually very small, it is preferable to use FCI_L alone. Example 10.1 is a comprehensive profitability analysis calculation using nondiscounted criteria.

Example 10.1

A new chemical plant is going to be built and will require the following capital investments (all figures are in $million):

Cost of land, *L* = $10.0

Total fixed capital investment, FCI_L = \$150.0

Fixed capital investment during year 1 = \$90.0

Fixed capital investment during year 2 = \$60.0

Plant startup at end of year 2

Working capital = \$30.0 at end of year 2

The sales revenues and costs of manufacturing are given below:

Yearly sales revenue (after startup), R = \$75.0/y

Cost of manufacturing excluding depreciation allowance (after startup), COM_d = \$30.0/y

Taxation rate, t = 45%

Salvage value of plant, S = \$10.0

Depreciation: Use 5-year MACRS.

Assume a project life of 10 years.

Calculate each nondiscounted profitability criterion given in this section for this plant.

Solution

The discrete and cumulative nondiscounted cash flows for each year are given in Table E10.1. Using these data, the cumulative cash flow diagram is drawn, as shown in Figure E10.1.

Table E10.1 Nondiscounted After-Tax Cash Flows for Example 10.1 (All Numbers in $\$10^6$)

End of Year (k)	Investment	d_k	$FCI_L - \Sigma d_k$	R	COM_d	$(R - COM - d_k) \times (1 - t) + d_k$	Cash Flow	Cumulative Cash Flow
0	(10)*	—	150.00	—	—	—	(10.00)	(10.00)
1	(90)	—	150.00	—	—	—	(90.00)	(100.00)
2	(60 + 30) = (90)	—	150.00	—	—	—	(90.00)	(190.00)
3	—	30.00	120.00	75	30	38.25	38.25	(151.75)
4	—	48.00	72.00	75	30	46.35	46.35	(105.40)
5	—	28.80	43.20	75	30	37.71	37.71	(67.69)
6	—	17.28	25.92	75	30	32.53	32.53	(35.16)
7	—	17.28	8.64	75	30	32.53	32.53	(2.64)
8	—	8.64	0.00	75	30	28.64	28.64	26.00
9	—	—	0.00	75	30	24.75	24.75	50.75
10	—	—	0.00	75	30	24.75	24.75	75.50
11	—	—	0.00	75	30	24.75	24.75	100.25
12	10 + 30 = 40	—	0.00	85	30	30.25	70.25	170.50

*Numbers in () are negative cash flows.

Nondiscounted Profitability Criteria

Payback Period (PBP)

Land + Working Capital = 10 + 30 = $\$40 \times 10^6$—find time after startup for which cumulative cash flow = $-\$40 \times 10^6$

PBP = 3 + (−67.69 + 40)/(−67.69 + 35.16) = **3.85 years**

Cumulative Cash Position (CCP) and Cumulative Cash Ratio (CCR)

CCP = $170.50 × 106 and CCR = Σ positive cash flows/Σ negative cash flows = (38.25 + 46.35 + 37.71 + … + 24.75 + 70.25) / (10 + 90 + 90) = **1.897**

Rate of Return on Investment (ROROI)

ROROI = (38.25 + 46.35 + 37.71 + … + 24.75 + 30.25)/10/150 – 1/10 = **0.114 or 11.4% p.a.**

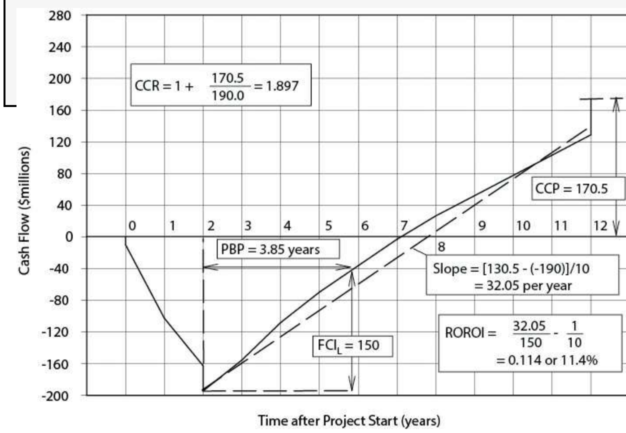


Figure E10.1 Cumulative Cash Flow Diagram for Nondiscounted After-Tax Cash Flows for Example 10.1

The method of evaluation for each of the criteria is given in Figure E10.1 and Table E10.1.

Payback Period (*PBP*) = 3.85 years

Cumulative Cash Position (*CCP*) = $\$170.5 \times 10^6$

Cumulative Cash Ratio (*CCR*) = 1.897

Rate of Return on Investment (*ROROI*) = 11.4%

All of these criteria indicate that the project cannot be eliminated as unprofitable. They all fail to take into account the time value of money that is necessary for a thorough measure of profitability. The effects of the time value of money on profitability are considered in the next section.

10.2.2 Discounted Profitability Criteria

The main difference between the nondiscounted and discounted criteria is that for the latter each of the yearly cash flows is discounted back to time zero. The resulting discounted cumulative cash flow diagram is then used to evaluate profitability. The three different types of criteria are: **Time Criterion.** The **discounted payback period (DPBP)** is defined in a manner similar to the nondiscounted version given above.

DPBP = Time required, after startup, to recover the fixed capital investment, FCI_L, required for the project, with all cash flows discounted back to time zero The project with the shortest discounted payback period is the most desirable.

Cash Criterion. The **discounted cumulative cash position,** more commonly known as the **net present value (NPV)** or **net present worth (NPW)** of the project, is defined as NPV = Cumulative discounted cash position at the end of the project

Again, the NPV of a project is greatly influenced by the level of fixed capital investment, and a better criterion for comparison of projects with different investment levels may be the **present value ratio (PVR):**

$$PVR = \frac{\text{Present Value of All Positive Cash Flows}}{\text{Present Value of All Negative Cash Flows}}$$

A present value ratio of unity for a project represents a break-even situation. Values greater than unity indicate profitable processes, whereas those less than unity represent unprofitable projects. Example 10.2 continues Example 10.1 using discounted profitability criteria.

Example 10.2

For the project described in Example 10.1, determine the following discounted profitability criteria:

1. Discounted payback period (DPBP)
2. Net present value (NPV)
3. Present value ratio (PVR)

   Assume a discount rate of 0.1 (10% p.a.).

Solution

The procedure used is similar to the one used for the nondiscounted evaluation shown in Example 10.1. The discounted cash flows replace actual cash flows. For the discounted case, all the cash flows in Table E10.1 must first be discounted back to the beginning of the project (time = 0). This is done by multiplying each cash flow by the discount factor ($P/F, i, n$), where n is the number of years after the start of the project. These discounted cash flows are shown along with the cumulative discounted cash flows in Table E10.2.

Table E10.2 Discounted Cash Flows for Example 10.2 (All Numbers in Millions of $)

End of Year	Nondiscounted Cash Flow	Discounted Cash Flow	Cumulative Discounted Cash Flow
0	(10.00)	(10)	(10.00)
1	(90.00)	(90)/1.1 = (81.82)	(91.82)
2	(90.00)	$(90)/1.1^2$ = (74.38)	(166.20)
3	38.25	$38.25/1.1^3$ = 28.74	(137.46)

4	46.35	$46.35/1.1^{4}$ = 31.66	(105.80)
5	37.71	$37.71/1.1^{5}$ = 23.41	(82.39)
6	32.53	$32.53/1.1^{6}$ = 18.36	(64.03)
7	32.53	$32.53/1.1^{7}$ = 16.69	(47.34)
8	28.64	$28.64/1.1^{8}$ = 13.36	(33.98)
9	24.75	$24.75/1.1^{9}$ = 10.50	(23.48)
10	24.75	$24.75/1.1^{10}$ = 9.54	(13.94)
11	24.75	$24.75/1.1^{11}$ = 8.67	(5.26)
12	70.25	$70.25/1.1^{12}$ = 22.38	17.12

Discounted Profitability Criteria

Discounted Payback Period (DPBP)

Discounted value of land + working capital = 10 + 30/1.12 = $\$34.8 \times 10^{6}$

Find time after startup when cumulative cash flow = $-\$34.8 \times 10^{6}$

DPBP = 5 + (−47.34 + 34.8)/(−47.34 + 33.98) = **5.94 y**

Net Present Value (NPV) and Present Value Ratio (PVR)

NPV = $17.12 × 106

PVR = Σ positive discounted cash flows/Σ negative discounted cash flows = (28.74 + 31.36 + 23.41 + ... + 22.38) / (10 + 81.82 + 74.38)

PVR = (183.31) / (166.2) = 1 + 17.12 / 166.2 = **1.10**

The cumulative discounted cash flows are shown on Figure E10.2, and the calculations are given in Table E10.2. From these sources the profitability criteria are given as

1. Discounted payback period (*DPBP*) = 5.94 years
2. Net present value (NPV) = $\$17.12 \times 10^{6}$
3. Present value ratio (*PVR*) = 1.10

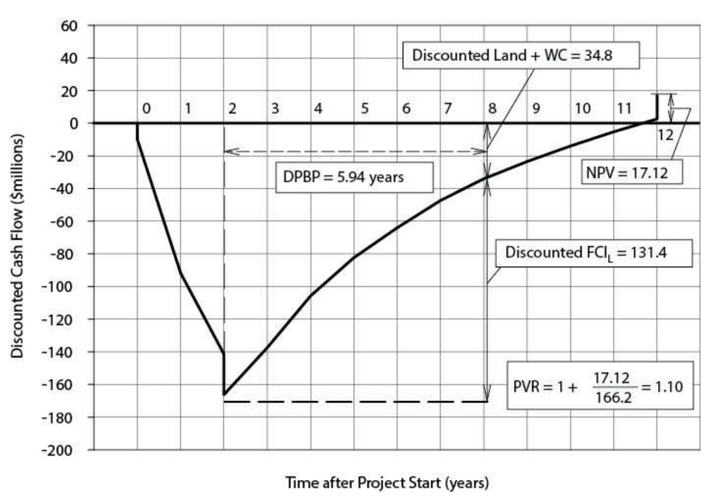


Figure E10.2 Cumulative Cash Flow Diagram for Discounted After-Tax Cash Flows for Example 10.2

These examples illustrate that there are significant effects of discounting the cash flows to account for the time value of money. From these results, the following observations may be made:

1. In terms of the time basis, the payback period increases as the discount rate increases. In the above examples, it increased from 3.85 to 5.94 years.
2. In terms of the cash basis, replacing the cash flow with the discounted cash flow decreases the value at the end of the project. In the above examples, it dropped from $170.5 to $17.12 million.
3. In terms of the cash ratios, discounting the cash flows gives a lower ratio. In the above examples, the ratio dropped from 1.897 to 1.10.

As the discount rate increases, all of the discounted profitability criteria are reduced.

Interest Rate Criterion. The **discounted cash flow rate of return (DCFROR)** is defined to be the interest rate at which all the cash flows must be discounted in order for the net present value of the project to be equal to zero. Thus, DCFROR = Interest or discount rate for which the net present value of the project is equal to zero

Therefore, the DCFROR represents the highest after-tax interest or discount rate at which the project can just break even.

For the discounted payback period and the net present value calculations, the question arises as to what interest rate should be used to discount the cash flows. This "internal" interest rate is usually determined by corporate management and represents the minimum rate of return that the company will accept for any new investment. Many factors influence the determination of this discount interest rate, and for current purposes, it is assumed that it is always given.

It should be noted that when evaluating the discounted cash flow rate of return, no interest rate is required because this is what is calculated. Clearly, if the DCFROR is greater than the internal discount rate, then the project is considered to be profitable. Use of DCFROR as a profitability criterion is illustrated in Example 10.3.

Example 10.3

For the problem presented in Examples 10.1 and 10.2, determine the discounted cash flow rate of return (DCFROR).

Solution

The NPVs for several different discount rates were calculated and the results are shown in Table E10.3. The value of the DCFROR is found at NPV = 0. Interpolating from Table E10.3 gives

$$\frac{(\text{DCFROR} - 12\%)}{(13\% - 12\%)} = \frac{(0 - 0.77)}{(-6.32 - 0.77)} = 0.109$$

Table E10.3 NPV for Project in Example 10.1 as a Function of Discount Rate

Interest or Discount Rate	NPV ($million)
0%	170.50
10%	17.12
12%	0.77
13%	−6.32
15%	−18.66
20%	−41.22

Therefore, DCFROR = 12 + 1(0.109) = 12.1%.

An alternate method for obtaining the DCFROR is to solve for the value of *i* in an implicit, nonlinear algebraic expression. This is illustrated in Example 10.4.

Figure 10.3 provides the cumulative discounted cash flow diagram for Example 10.3 for several discount factors. It shows the effect of changing discount factors on the profitability and shape of the curves. It includes a curve for the DCFROR found in Example 10.3. For this case, it can be seen that the NPV for the project is zero. In Example 10.3, if the acceptable rate of return for a company were set at 20%, then the project would not be considered an acceptable investment. This is indicated by a negative NPV for *i* = 20%.

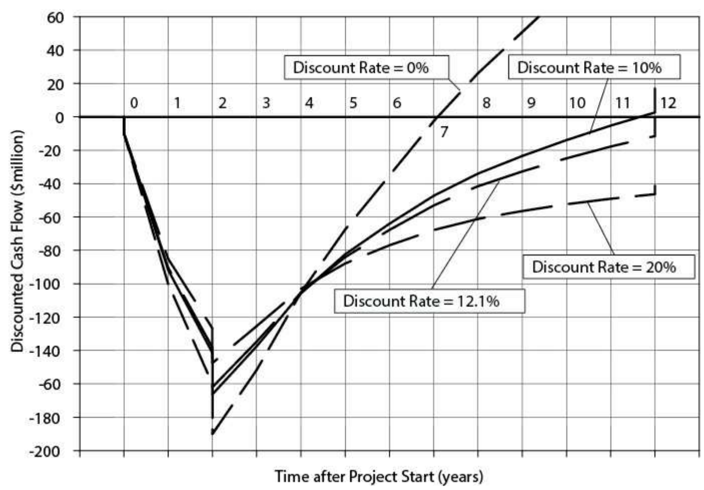


Figure 10.3 Discounted Cumulative Cash Flow Diagrams Using Different Discount Rates for Example 10.3

Each method described above used to gauge the profitability of a project has advantages and disadvantages. For projects having a short life and small discount factors, the effect of discounting is small, and nondiscounted criteria may be used to give an accurate measure of profitability. However, it is fair to say that for large projects involving many millions of dollars of capital investment, discounting techniques should always be used.

Because all the above techniques are commonly used in practice, familiarity with and being able to use each technique are important.

10.3 COMPARING SEVERAL LARGE PROJECTS: INCREMENTAL ECONOMIC ANALYSIS

In this section, comparison and selection among investment alternatives are discussed. When comparing project investments, the DCFROR tells how efficiently money is being used. When using this criterion, it should be noted that the higher the DCFROR, the more attractive is the individual investment. However, when comparing investment alternatives, it may be better to choose a project that does not have the highest DCFROR. The rationale for comparing projects and choosing the most attractive alternative is discussed in this section.

In order to make a valid decision regarding alternative investments (projects), it is necessary to know a baseline rate of return that must be attained in order for an investment to be attractive. A company that is considering whether to invest in a new project always has the option to reject all alternatives offered and invest the cash (or resources) elsewhere. The baseline or benchmark investment rate is related to these alternative investment opportunities, such as investing in the stock market. Incremental economic analysis is illustrated in Examples 10.4 and 10.5.

Example 10.4

A company is seeking to invest approximately $\$120 \times 10^6$ in new projects. After extensive research and preliminary design work, three projects have emerged as candidates for construction. The minimum acceptable internal discount (interest) rate, after tax, has been set at 10%. The after-tax cash-flow information for the three projects using a ten-year operating life is as follows (values in $million):

	Initial Investment ($million)	After-Tax Cash $k = 1$	Flow in Year k $k = 2–10$
Project A	$60	$10	$12
Project B	$120	$22	$22
Project C	$100	$12	$20

For this example it is assumed that the costs of land, working capital, and salvage are zero. Furthermore, it is assumed that the initial investment occurs at time = 0, and the yearly annual cash flows occur at the end of each of the ten years of plant operation.

Determine the following:

1. The NPV for each project
2. The DCFROR for each project

Solution

For Project A,

$$\text{NPV} = -\$60 + (\$10)(P/F, 0.10, 1) + (\$12)(P/A, 0.10, 9)(P/F, 0.10, 1)$$
$$= -\$60 + \frac{(\$10)}{1.1} + (\$12)\frac{1.1^9 - 1}{(0.1)(1.1^9)}\frac{1}{1.1} = \$11.9$$

The DCFROR is the value of i that results in NPV = 0:

$$\text{NPV} = 0 = -\$60 + (\$10)(P/F, i, 1) + (\$12)(P/A, i, 9)(P/F, i, 1)$$

Solving for i yields i = DCFROR = 14.3%.

Values obtained for NPV and DCFROR are as follows:

	NPV (i = 10%)	DCFROR
Project A	11.9	14.3%
Project B	15.2	12.9%
Project C	15.6	13.3%

Note: Projects A, B, and C are mutually exclusive because investment cannot be made in more than one of them, due to the cap of $\$120 \times 10^6$. The analysis that follows is limited to projects of this type. For the case when projects are not mutually exclusive, the analysis becomes somewhat more involved and is not covered here.

Although all the projects in Example 10.4 showed a positive NPV and a DCFROR of more than 10%, at this point it is not clear how to select the most attractive option with this information. It will be seen later that the choice of the project with the highest NPV will be the most attractive. However, consider the following alternative analysis. If Project B is selected, a total of $\$120 \times 10^6$ is invested and yields 12.9%, whereas the selection of Project A yields 14.3% on the $\$60 \times 10^6$ invested. To compare these two options, a situation in which the same amount is invested in both cases would have to be considered. In Project A, this would mean that $\$60 \times 10^6$ is invested in the project and the remaining $\$60 \times 10^6$ is invested elsewhere, whereas in Project B, a total of $\$120 \times 10^6$ is invested in the project.

It is necessary in the analysis to be sure that the last dollar invested earns at least 10%. To do this, an incremental analysis must be performed on the cash flows to establish that at least

10% is made on each additional increment of money invested in the project.

Example 10.5

This is a continuation of Example 10.4.

1. Determine the NPV and the DCFROR for each increment of investment.
2. Recommend the best option.

Solution

1. First, Project A and Project C are compared, since Project C is the next larger investment:
   Incremental investment is $\$40 \times 10^6 = (\$100 - \$60) \times 10^6$.
   Incremental cash flow for $i = 1$ is $\$2 \times 10^6/\text{y} = (\$12/\text{y} - \$10/\text{y}) \times 10^6$.
   Incremental cash flow for $i = 2$ to 10 is $\$8 \times 10^6/\text{y} = (\$20/\text{y} - \$12/\text{y}) \times 10^6$.

   $$\text{NPV} = -\$40 \times 10^6 + (\$2 \times 10^6)(P/F, 0.10, 1) + (\$8 \times 10^6)(P/A, 0.10, 9)(P/F, 0.10, 1)\ \text{NPV} = \$3.7 \times 10^6$$

   Setting NPV = 0 yields DCFROR = 0.119 (11.9%), which is acceptable.

   Since Project C is acceptable, Project C and Project B are compared:

   Incremental investment is $\$20 \times 10^6 = (\$120 - \$100) \times 10^6$.
   Incremental cash flow for $i = 1$ is $\$10 \times 10^6/\text{y} = (\$22/\text{y} - \$12/\text{y}) \times 10^6$.
   Incremental cash flow for $i = 2$ to 10 is $\$2 \times 10^6/\text{y} = (\$22/\text{y} - \$20/\text{y}) \times 10^6$.

   $$\text{NPV} = -\$0.4 \times 10^6 \text{ and DCFROR} = 0.094\ (9.4\%)$$

2. It is recommended to move ahead on Project C.

From Example 10.5, it is clear that the rate of return on the $\$20 \times 10^6$ incremental investment required to go from Project C to Project B did not return the 10% required and gave a negative NPV.

The information from Example 10.4 shows that an overall return on investment of more than 10% is obtained for each of the three projects. However, the correct choice, Project C, also has the highest NPV using a discount rate of 10%, and it is this criterion that should be used to compare alternatives.

When carrying out an incremental investment analysis on projects that are mutually exclusive, the following four-step algorithm is recommended: **Step 1:** Establish the minimum acceptable rate of return on investment for such projects.

> When comparing mutually exclusive investment alternatives, choose the alternative with the greatest positive net present value.

Step 2: Calculate the NPV for each project using the interest rate from Step 1.

Step 3: Eliminate all projects with negative NPV values.

Step 4: Of the remaining projects, select the project with the highest NPV.

10.4 ESTABLISHING ACCEPTABLE RETURNS FROM INVESTMENTS: THE CONCEPT OF RISK

Most comparisons of profitability will involve the rate of return of an investment. Company management usually provides several **benchmarks** or **hurdle rates** for acceptable rates of return that must be used in comparing alternatives.

A company vice president (VP) has been asked to recommend one of the following two alternatives to pursue.

Option 1: A new product is to be produced that has never been made before on a large scale. Pilot plant runs have been made and the products sent to potential customers. Many of these customers have expressed an interest in the product but need more material to evaluate it fully. The calculated return on the investment for this new plant is 33%.

Option 2: A second plant is to be built in another region of the country to meet increasing demand in the region. The company has a dominant market position for this product. The new facility would be similar to other plants. It would involve more computer control, and attention will be paid to meeting pending changes in environmental regulations. The rate of return is calculated to be 12%.

The recommendation of the VP and the justifications are given below.

Items that favor Option 1 if pursued:

- High return on the investment
- Opens new product possibilities

Items that favor Option 2:

- The market position for Option 2 is well established. The market for the new product has not been fully established.
- The manufacturing costs are well known for Option 2 but are uncertain for the new process because only estimates are available.
- Transportation costs will be less than current values due to the proximity of plant.
- The technology used in Option 2 is mature and well known. For the new process, there is no guarantee that it will work.

The closing statement from the VP included the following summary:

> *"We have little choice but to expand our established product line. If we fail to build these new production facilities, our competitors are likely to build a new plant in the region to meet the increasing demand. They could undercut our regional prices, and this would put at risk our market share and dominant market position in the region."*

Clearly, the high return on investment for Option 1 was associated with a high risk. This is usually the case. There are often additional business reasons that must be considered prior to making the final decision. The concern for lost market position is a serious one and weighs heavily in any decision. The relatively low return on investment of 12% given in this example would probably not be very attractive had it not been for this concern. It is the job of company management to weigh all of these factors, along with the rate of return, in order to make the final decision.

In this chapter, the terms *internal interest/discount rates* and *internal rates of return* are used. This deals with benchmark interest rates that are to be used to make profitability evaluations. There are likely to be different values that reflect dissimilar conditions of risk—that is, the value for mature technology would differ from that for unproven technology. For example, the internal rate of return for mature technology might be set at 12%, whereas that for very new technology might be set at 40%. Using these values the decision by the VP given above seems more reasonable. The analysis of risk is considered in Section 10.7.

10.5 EVALUATION OF EQUIPMENT ALTERNATIVES

Often during the design phases of a project, it will be necessary to evaluate different equipment options. Each alternative piece of equipment performs the same process function. However, the capital cost, operating cost, and equipment life may be different for each, and the best choice must be determined using some economic criterion.

Clearly, if there are two pieces of equipment, each with the same expected operating life that can perform the desired function with the same operating cost, then *common sense* suggests choosing the *less expensive alternative!* When the expected life and operating expenses vary, the selection becomes more difficult. Techniques available to make the selection are discussed in this section.

10.5.1 Equipment with the Same Expected Operating Lives

When the operating costs and initial investments are different but the equipment lives are the same, then the choice should be made based on NPV. The choice with the least negative NPV will be the best choice. Examples 10.6 and 10.7 illustrate evaluation of equipment alternatives.

Example 10.6

In the final design stage of a project, the question has arisen as to whether to use a water-cooled exchanger or an air-cooled exchanger in the overhead condenser loop of a distillation tower. The information available on the two pieces of equipment is provided as follows:

	Initial Investment	Yearly Operating Cost
Air-cooled	$23,000	$1200
Water-cooled	$12,000	$3300

Both pieces of equipment have service lives of 12 years. For an internal rate of return of 8% p.a., which piece of equipment represents the better choice?

Solution

The NPV for each exchanger is evaluated as follows:

NPV = −Initial Investment + (Operating Cost)(P/A, 0.08, 12)

	NPV
Air-cooled	−$32,040
Water-cooled	−$36,870

The air-cooled exchanger represents the better choice.

Despite the higher capital investment for the air-cooled exchanger in Example 10.6, it was the recommended alternative. The lower operating cost more than compensated for the higher initial investment.

10.5.2 Equipment with Different Expected Operating Lives

When process units have different expected operating lives, care is required in determining the best choice. In terms of expected equipment life, it is assumed that this is less than the expected working life of the plant. Therefore, during the normal operating life of the plant, it can be expected that the equipment will be replaced at least once. This requires that different profitability criteria be applied. Three commonly used methods are presented to evaluate this situation. (The effect of inflation is not considered in these methods.) All methods consider both the capital and operating cost in minimizing expenses, thereby maximizing profits.

Capitalized Cost Method. In this method, a fund is established for each piece of equipment to be compared. This fund provides the amount of cash that would be needed to

1. Purchase the equipment initially

2. Replace it at the end of its life
3. Continue replacing it forever

The size of the initial fund and the logic behind the capitalized cost method are illustrated in Figure 10.4.

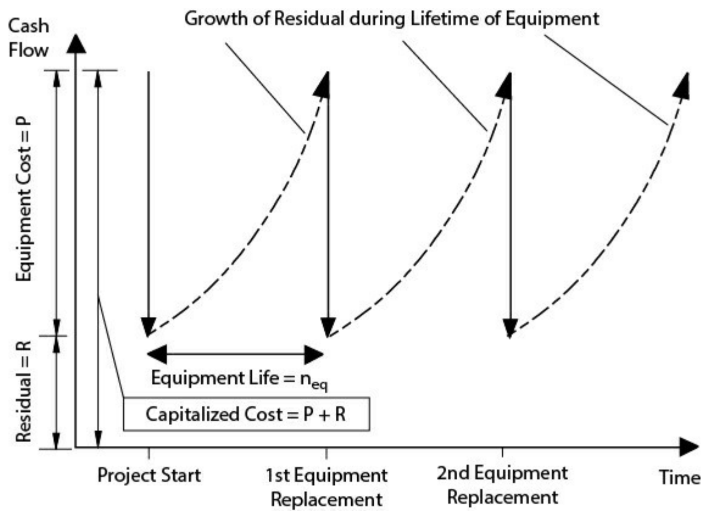


Figure 10.4 An Illustration of the Capitalized Cost Method for the Analysis of Equipment Alternatives

From Figure 10.4, it can be seen that if the equipment replacement cost is *P*, then the total fund set aside (called the **capitalized cost**) is $P + R$, where *R* is termed the **residual.** The purpose of this residual is to earn sufficient interest during the life of the equipment to pay for its replacement. At the end of the equipment life, n_{eq}, the amount of interest earned is *P*, the equipment replacement cost. As Figure 10.4 shows, replacing the equipment every time it wears out may continue. Referring to Figure 10.4, the equation is developed for the capitalized cost defined as $(P + R)$:

$$R(1+i)^{n_{eq}} - R = P$$

and

$$\text{Capitalized Cost} = P + R = P + \frac{P}{(1+i)^{n_{eq}} - 1} = P\left[\frac{(1+i)^{n_{eq}}}{(1+i)^{n_{eq}} - 1}\right] \quad (10.1)$$

The term in square brackets in Equation (10.1) is commonly referred to as the **capitalized cost factor.**

The capitalized cost obtained from Equation (10.1) does not include the operating cost and is useful in comparing alternatives only when the operating costs of the alternatives are the same. When operating costs vary, it is necessary to capitalize the operating cost. An equivalent capitalized operating cost that converts the operating cost into an equivalent capital cost is added to the capitalized cost calculated from Equation (10.1) to provide the **equivalent capitalized cost (ECC):**

$$\text{Equivalent Capitalized Cost} = \text{Capitalized Cost} + \text{Capitalized OperatingCost}$$

$$\text{Equivalent Capitalized Cost} = \left[\frac{P(1+i)^{n_{eq}} + YOC(F/A,i,n_{eq})}{(1+i)^{n_{eq}}-1}\right] \quad (10.2)$$

This cost considers both the capital cost of equipment and the **yearly operating cost (YOC)** needed to compare alternatives. The extra terms in Equation (10.2) represent the effect of taking the yearly cash flows for operating costs from the residual, *R*.

By using Equation (10.1) or (10.2), it is possible to account correctly for the different operating lives of the equipment by calculating an effective capitalized cost for the equipment and operating cost. Example 10.7 illustrates the use of these equations.

Example 10.7

During the design of a new project, a decision must be made regarding which type of pump should be used for a corrosive service. The options are as follows:

	Capital Cost	Operating Cost (per year)	Equipment Life (years)
Carbon steel pump	$8000	$1800	4
Stainless steel pump	$16,000	$1600	7

Assume a discount rate of 8% p.a.

Solution

Using Equation (10.2) for the carbon steel pump,

$$\text{Capitalized Cost} = \frac{(8000)\,(1.08)^4 + (1800)\,\frac{[1.08^4-1]}{0.08}}{1.08^4 - 1} = \$52,700$$

For the stainless steel pump,

$$\text{Capitalized Cost} = \frac{(16,000)(1.08)^7 + (1600)\frac{[1.08^7-1]}{0.08}}{1.08^7 - 1} = \$58,400$$

The carbon steel pump is recommended because it has the lower capitalized cost.

In Example 10.7, the stainless steel pump costs twice as much as the carbon steel pump and, because of its superior resistance to corrosion, will last nearly twice as long. In addition, the operating cost for the stainless steel pump is lower due to lower maintenance costs. In spite of these advantages, the carbon steel pump was still judged to offer a cost advantage.

Equivalent Annual Operating Cost (EAOC) Method. In the previous method, both capital cost and yearly operating costs were lumped into a single cash fund or equivalent cash amount. An alternative method is to **amortize** (spread out) the capital cost of the equipment over the operating life to establish a yearly cost. This is added to the operating cost to yield the EAOC.

Figure 10.5 illustrates the principles behind this method. From Figure 10.5, it can be seen that the cost of the initial purchase will be spread out over the operating life of the equipment. The EAOC is expressed by Equation (10.3):

$$EAOC = (\text{Capital Investment})(A/P, i, n_{eq}) + YOC \quad (10.3)$$

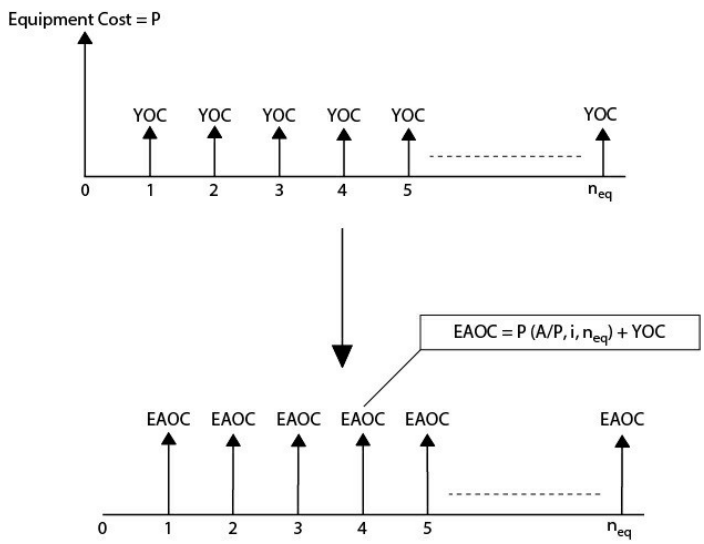


Figure 10.5 Cash Flow Diagrams Illustrating the Concept of Equivalent Annual Operating Cost

The EAOC method can be understood in terms of the everyday example of comparing new car alternatives. The EAOC can be used to determine whether a higher capital investment, for example, buying a hybrid vehicle, is worthwhile, based on the anticipated savings in the yearly operating cost (reduced fuel consumption).

Example 10.8 illustrates this method for two pumps.

Example 10.8

Compare the stainless steel and carbon steel pumps in Example 10.7 using the EAOC method.

Solution

For the carbon steel pump,

$$EAOC = \frac{(8000)(0.08)(1.08)^4}{1.08^4 - 1} + 1800$$
$$= \$4220 \text{per year} (\text{better option})$$

For the stainless steel pump,

$$EAOC = \frac{(16,000)(0.08)(1.08)^7}{1.08^7 - 1} + 1600$$
$$= \$4670 \text{per year}$$

The carbon steel pump is shown to be the preferred equipment using the EAOC method, as it was in Example 10.7 using the ECC method.

Common Denominator Method. Another method for comparing equipment with unequal operating lives is the **common denominator method.** This method is illustrated in Figure 10.6, in which two pieces of equipment with operating lives of n and m years are to be compared. This comparison is done over a period of nm years during which the first piece of equipment will need m replacements and the second will require n replacements. Each piece of equipment has an integer number of replacements, and the time over which the comparison is made is the same for both pieces of equipment. For these reasons the comparison can be made using the net present value of each alternative. In general, an integer number of replacements can be made for both pieces of equipment in a time N, where N is the smallest number into which m and n are both exactly divisible; that is, N is the common denominator. Example 10.9 illustrates this method.

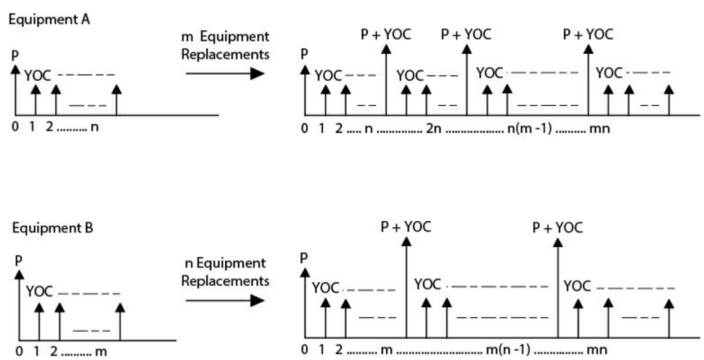


Figure 10.6 An Illustration of the Common Denominator Method for the Analysis of Equipment Alternatives

Example 10.9

Compare the two pumps given in Example 10.7 using the common denominator method. The discrete cash flow diagrams for the two pumps are shown in Figure E10.9. The minimum time over which the comparison can be made is 4(7) = 28 years.

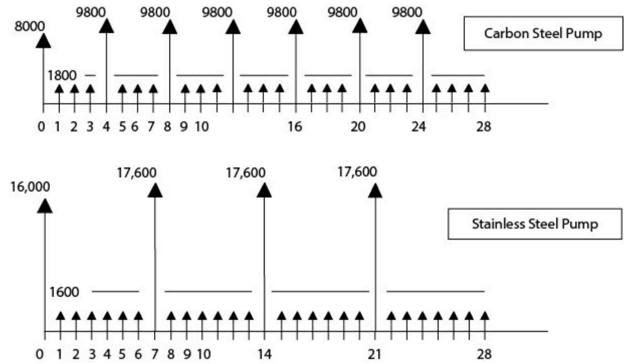


Figure E10.9 Cash Flow Diagrams for the Common Denominator Method Used in Example 10.9

Solution

NPV for the carbon steel pump:

NPV	$= -(\$8000)(1 + 1.08^{-4} + 1.08^{-8} + 1.08^{-12} + 1.08^{-16} + 1.08^{-20} + 1.08^{-24})$
	$-(\$1800)(P/A, 0.08, 28)$
	$= -\$46{,}580$ (better option)

NPV for the stainless steel pump:

$$\text{NPV} = -(\$16{,}000)(1 + 1.08^{-7} + 1.08^{-14} + 1.08^{-21}) - (\$1600)(P/A, 0.08, 28) = -\$51{,}643$$

The carbon steel pump has a less negative NPV and is recommended.

As found for the previous two methods, the common denominator method favors the carbon steel pump.

Choice of Methods. Because all three methods of comparison correctly take into account the time value of money, the results of all the methods are equivalent. In most problems, the common denominator method becomes unwieldy. The use of the EAOC or the capitalized cost methods are favored for these calculations.

10.6 INCREMENTAL ANALYSIS FOR RETROFITTING FACILITIES

This topic involves profitability criteria used for analyzing situations where a piece of equipment is added to an existing facility. The purpose of adding the equipment is to improve the profitability of the process. Such improvements are often referred to as **retrofitting.** Such retrofits may be extensive—requiring millions of dollars of investment—or small, requiring an investment of only a few thousand dollars.

The decisions involved in retrofitting projects may be of the discrete type, the continuous type, or a combination of both. An example of a discrete decision is whether to add an on-line monitoring and control system to a wastewater stream. The decision is a simple yes or no. An example of a continuous decision is to determine what size of heat recovery system should be added to an existing process heater to improve fuel efficiency. This type of decision would involve sizing the optimum heat exchanger, where the variable of interest (heat-exchanger area) is continuous.

Because retrofit projects are carried out on existing operating plants, it becomes necessary to identify all of the costs and savings associated with the retrofit. When comparing alternative schemes, the focus of attention is on the profitability of the incremental investment required. Simple, discrete choices will be considered in this section. The problem of optimizing a continuous variable is covered in Chapter 14.

The initial step in an incremental analysis of competing alternatives is to identify the potential alternatives to be considered and to specify the increments over which the analysis is to be performed. The first step is to rank the available alternatives by the magnitude of the capital cost. The alternatives will be identified as $A_1, A_2, \ldots, A_n$. There are n possible alternatives. The first alternative, A_1, which is always available, is the "do nothing" option. It requires no capital cost (and achieves no savings). For each of the available alternatives, the project cost (capital cost), *PC*, and the yearly savings generated (yearly cash flow), *YS*, must be known.

For larger retrofit projects, discounted profitability criteria should be used. The algorithm to compare alternatives using discounted cash flows follows the same four-step method outlined in Section 10.3. For small retrofit projects, nondiscounted criteria may often be sufficiently accurate for comparing alternatives. Both types of criteria are discussed in the next sections.

10.6.1 Nondiscounted Methods for Incremental Analysis

For nondiscounted analyses, two methods are provided below.

1. Rate of Return on Incremental Investment (ROROII)

$$ROROII = \frac{\text{Incremental Yearly Savings}}{\text{Incremental Investment}}$$

2. Incremental Payback Period (IPBP)

$$IPBP = \frac{\text{Incremental Investment}}{\text{Incremental Yearly Savings}}$$

It is observed that these two parameters are reciprocals of each other.

Examples 10.10–10.12 illustrate the method of comparison of projects using these two criteria.

Example 10.10

A circulating heating loop for an endothermic reactor has been in operation for several years. Due to an oversight in the design phase, a certain portion of the heating loop piping was left uninsulated. The consequence is a significant energy loss. Two types of insulation can be used to reduce the heat loss. They are both available in two thicknesses. The estimated cost of the insulation and the estimated yearly savings in energy costs are given in Table E10.10. (The ranking has been added to the alternatives and is based on increasing project cost.)

Table E10.10 Rankings of Alternative Insulations for Example 10.10

Ranking (Option #)	Alternative Insulation	Project Cost (*PC*)	Yearly Savings Generated by Project (*YS*)
1	No Insulation	0	0
2	B-1" Thick	$3000	$1400
3	B-2" Thick	$5000	$1900
4	A-1" Thick	$6000	$2000
5	A-2" Thick	$9700	$2400

Assume an acceptable internal rate of return for a nondiscounted profitability analysis to be 15% (0.15).

1. For the four types of insulation determine the rate of return on incremental investment (ROROII) and the incremental payback period (IPBP).
2. Determine the value of the incremental payback period equivalent to the 15% internal rate of return.

Solution

1. Evaluation of ROROII and IPBP

Option #-Option 1	*ROROII*	*IPBP* (years)
2-1	$1400/$3000 = 0.47 (47%)	$3000/$1400 = 2.1
3-1	$1900/$5000 = 0.38 (38%)	$5000/$1900 = 2.6
4-1	$2000/$6000 = 0.33 (33%)	$6000/$2000 = 3.0
5-1	$2400/$9700 = 0.25 (25%)	$9700/$2400 = 4.0

2. $IPBP = 1/(ROROII) = 1/0.15 = 6.67$ y

Note that in Part (a) of Example 10.10, the incremental investment and savings are given by the difference between installing the insulation and doing nothing. All of the investments considered in Example 10.10 satisfied the internal benchmark for investment of 15%, which means that the do-nothing option (Option 1) can be discarded. However, which of the remaining options is the best can be determined only by using pairwise comparisons.

Example 10.11

Which of the options in Example 10.10 is the best based on the nondiscounted ROROII of 15%?

Solution

Step 1: Choose Option 2 as the base case, because it has the lowest capital investment.

Step 2: Evaluate incremental investment and incremental savings in going from the base case to the case with the next higher capital investment, Option 3.

Incremental Investment = ($5000 − $3000) = $2000

Incremental Savings = ($1900/y − $1400/y) = $500/y

$ROROII = 500/2000 = 0.4$, or 40%/y

Step 3: Because the result of Step 2 gives an $ROROII > 15\%$, Option 3 is used as the base case and is compared with the option with the next higher capital investment, Option 4.

Incremental Investment = ($6000 − $5000) = $1000
Incremental Savings = ($2000/y − $1900/y) = $100/y
ROROII = 100/1000 = 0.1 or 10%/y
Step 4: Because the result of Step 3 gives an *ROROII* < 0.15, Option 4 is rejected and Option 3 is compared with the option with the next higher capital investment, Option 5.
Incremental Investment = ($9700 − $5000) = $4700
Incremental Savings = ($2400/y − $1900/y) = $500/y
ROROII = 500/4700 = 0.106, or 10.6%/y
Step 5: Again, the ROROII from Step 4 is less than 15%, and hence Option 5 is rejected. Because Option 3 (Insulation B-2" thick) is the current base case and no more comparisons remain, Option 3 is accepted as the "best option."
It is important to note that in Example 10.11 Options 4 and 5 are rejected even though they give *ROROII* > 15% when compared with the do-nothing option (see Example 10.10). The key here is that in going from Option 3 to either Option 4 or 5 the incremental investment loses money, that is, *ROROII* < 15%.
Example 10.12
Repeat the comparison of options in Example 10.10 using a nondiscounted incremental payback period of 6.67 years.
Solution
The steps are similar to those used in Example 10.11 and are given below without further explanation.
Step 1: (Option 3 – Option 2) *IPBP* = 2000/500 = 4 years < 6.67
Reject Option 2; Option 3 becomes the base case.
Step 2: (Option 4 – Option 3) *IPBP* = 1000/10 = 10 years > 6.67
Reject Option 4.
Step 3: (Option 5 – Option 3) *IPBP* = 4700/500 = 9.4 years > 6.67
Reject Option 5.
Option 3 is the best option.

10.6.2 Discounted Methods for Incremental Analysis

Incremental analyses taking into account the time value of money should always be used when large capital investments are being considered. Comparisons may be made either by discounting the operating costs to yield an equivalent capital investment or by amortizing the initial investment to give an equivalent annual operating cost. Both techniques are considered in this section, where the effects of depreciation and taxation are ignored in order to keep the analysis simple. However, it is an easy matter to take these effects into account.
Capital Cost Methods. The incremental net present value (INPV) for a project is given by

$$INPV = -PC + YS\,(P/A, i, n) \qquad (10.4)$$

When comparing investment options, a given case option will always be compared with a do-nothing option. Thus, these comparisons may be considered as incremental investments. In order to use INPV, it is necessary to know the internal discount rate and the time over which the comparison is to be made. This method is illustrated in Example 10.13.
Example 10.13
Based on the information provided in Example 10.10 for an acceptable internal interest rate of 15% and time n = 5 y, determine the most attractive alternative, using the INPV criterion to compare options.
Solution
For i = 0.15 and n = 5 the value for $(P/A, i, n)$ = 3.352. (See Equation [9.14].)
Equation (10.4) becomes
$INPV = -PC + 3.352\,YS$

Option	***INPV* ($)**
2-1	1693
3-1	1369
4-1	704
5-1	–1655

From the results above, it is clear that Options 2, 3, and 4 are all potentially profitable as *INPV* > 0. However, the best option is Option 2 because it has the highest INPV when compared with the do-nothing case, Option 1. Note that other pairwise comparisons are unnecessary. The option that yields the highest INPV is chosen. The reason that the INPV gives the best option directly is because by knowing i and n, each dollar of incremental investment is correctly accounted for in the calculation of INPV. Thus, if the incremental investment in going from Option A to Option B is profitable, then the INPV will be greater for Option B and vice versa. It should also be pointed out that by using discounting techniques, the best option has changed from

Option 3 (in Example 10.11) to Option 2.

Operating Cost Methods. In the previous section, yearly savings were converted to an equivalent present value using the present value of an annuity, and this was measured against the capital cost. An alternative method is to convert all the investments to annual costs using the capital recovery factor and measure them against the yearly savings.

The needed relationship is developed from Equation (10.4), giving

$INPV/(P/A, i, n) = -PC/(P/A, i, n) + YS$

It can be seen that the capital recovery factor $(A/P,i,n)$ is the reciprocal of the present worth factor $(P/A,i,n)$. Substituting this relationship and multiplying by −1 gives $-(INPV)(A/P, i, n) = (PC)(A/P, i, n) - YS$

The term on the left is identified as the Equivalent Annual Operating Cost (EAOC). Thus,

$$EAOC = (PC)\,(A/P, i, n) - YS \qquad (10.5)$$

When an acceptable rate for i and n is substituted, a negative EAOC indicates the investment is acceptable (because a negative cost is the same as a positive savings). Use of EAOC is demonstrated in Example 10.14.

Example 10.14

Repeat Example 10.13 using *EAOC* in place of NPV.

Solution

For $i = 0.15$ and $n = 5$ the value for $(A/P, i, n) = 1/3.352$.

Equation (10.5) becomes

$EAOC = PC/3.352 - YS$

Option	***EAOC* ($/y)**
2–1	−505
3–1	−408
4–1	−210
5–1	494

The best alternative is Option 2 because it has the most negative EAOC.

10.7 EVALUATION OF RISK IN EVALUATING PROFITABILITY

In this section, the concept of risk in the evaluation of profitability is introduced, and the techniques to quantify it are illustrated. Until now, it has been assumed that the financial analysis is essentially deterministic—that is, all factors are known with absolute certainty. Recalling discussions in Chapter 7 regarding the relative error associated with capital cost estimates, it should not be surprising that many of the costs and parameters used in evaluating the profitability of a chemical process are estimates that are subject to error. In fact, nearly all of these factors are subject to change throughout the life of the chemical plant. The question then is not, “Do these parameters change?” but rather, “By how much do they change?” In Table 10.1, due to Humphreys [1], ranges of expected variations for factors that affect the prediction and forecasting of profitability are given.

Table 10.1 Range of Variation of Factors Affecting the Profitability of a Chemical Process

Factor in Profitability Analysis	**Probable Variation from Forecasts over 10-Year Plant Life, %**
Cost of fixed capital investment*	−10 to +25
Construction time	−5 to +50
Startup costs and time	−10 to +100
Sales volume	−50 to +150
Price of product	−50 to +20
Plant replacement and maintenance costs	−10 to +100
Income tax rate	−5 to +15
Inflation rates	−10 to +100
Interest rates	−50 to + 50
Working capital	−20 to +50
Raw material availability and price	−25 to +50
Salvage value	−100 to +10
Profit	−100 to +10

*For capital cost estimations using CAPCOST, a more realistic range is –20 to +30%.

(From *Jelen’s Cost and Optimization Engineering*, 3rd ed., by K. K. Humphreys (1991), reproduced by permission of the McGraw-Hill Companies, Inc.)

The most important variable in Table 10.1 is sales volume, with the price of product and raw material being a close second. Clearly, if market forces were such that it was possible to sell (and hence produce) only 50% of the originally estimated amount of product, then profitability would be affected greatly. Indeed, the process would quite possibly be unprofitable. The problem is that projections of how the variables will vary over the life of the plant are difficult (and sometime impossible) to estimate. Nevertheless, experienced cost estimators often have a feel for the variability of some of these parameters. In addition, marketing and financial specialists within large companies have expertise in forecasting trends in product demand, product price, and raw material costs. In the next section, the effect that supply and demand have on the sales price of a product is investigated. Following this, methods to quantify risk and to predict the range of profitability that can be expected from a process, when uncertainty exists in some of the profitability parameters listed in Table 10.1, will be discussed.

10.7.1 Forecasting Uncertainty in Chemical Processes

In order to be able to predict the way in which the factors in Table 10.1 vary, it is necessary to take historical data along with information about new developments to formulate a model to predict trends in key economic parameters over the projected life of a process. This prediction process is often referred to as forecasting and is, in general, a very inexact science. The purpose of this section is to introduce some concepts that must be considered when quantifying economic projections. A detailed description of the art of economic forecasting is way beyond the scope of this text. Instead, the basic concepts and factors influencing economic parameters are introduced.

Supply and Demand Concepts in Chemical Markets. Economists use microeconomic theory [2] to describe how changes in the supply of and demand for a given product are affected by changes in the market. Only the most basic supply and demand curves, shown in Figure 10.7, are considered here.

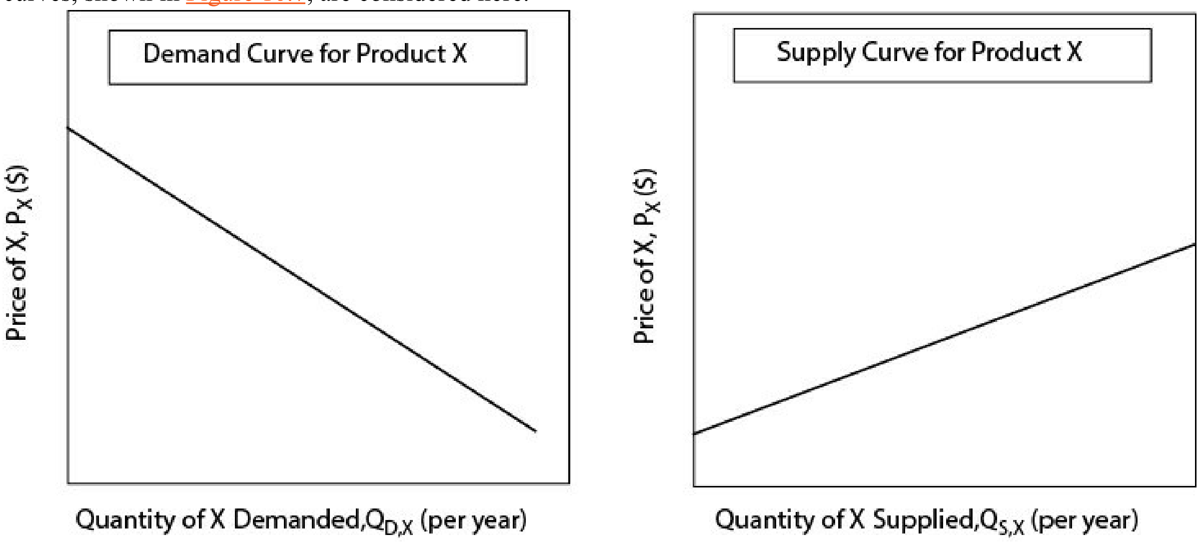


Figure 10.7 Simple Supply and Demand Curves for Product X

The demand curve (on the left) slopes downward and shows the general trend that as the price for commodity X decreases, the demand increases. With very few exceptions, this is always true. Examples of chemical products following this trend are numerous; for example, as the price of gasoline, polyethylene, or fertilizer drops, the demand for these goods increases (all other factors remaining constant). The supply curve (shown on the right) slopes upward and shows the trend that as the price rises, the amount of product X that manufacturers are willing to produce increases. The slope of the supply curve is often positive but may also be negative depending on the product. For most chemical products, it can be assumed that the slope is positive, and with all other factors remaining constant, the quantity supplied increases as the price for the product increases. Unlike physical laws that govern thermodynamics, heat transfer, and so on, these trends are not absolute. Instead, these trends reflect human nature relating to buying and selling of goods.

When market forces are in equilibrium, the supply and demand for a given product are balanced, and the equilibrium price (P_{eq}) is determined by the intersection of the supply and demand curves, as shown in Figure 10.8.

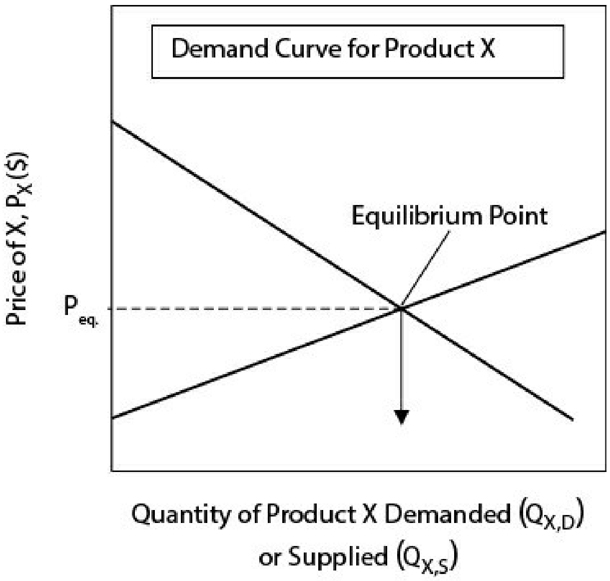


Figure 10.8 Illustration of Market Equilibrium for Product X

There are many factors that can affect the market for product X. Indeed, a market may not be in equilibrium, and, in this case, the market price must be determined in terms of rate equations as opposed to equilibrium relationships. However, for the sake of this simplified discussion, it will be assumed that market equilibrium is always reached. If something changes in the market, either the supply or demand curve (or both) will shift, and a new equilibrium point will be reached. As an example, consider the situation when a large new plant that produces X comes on line. Assuming that nothing else in the market changes, the supply curve will be shifted downward and to the right, which will lead to a lower equilibrium price. This situation is illustrated in Figure 10.9. The intersection of the demand curve and the new supply curve gives rise to the new equilibrium price, $P_{eq,2}$, which is lower than the original equilibrium price, $P_{eq,1}$. The magnitude of the decrease in the equilibrium price depends on the magnitude of the downward shift in the supply curve. If the new plant is large compared with the total current manufacturing capacity for product X, then the decrease in the equilibrium price will be correspondingly large. If this decrease in price is not taken into account in the economic analysis, the projected profitability of the new project will be overestimated, and the decision to invest might be made when the correct decision would be to abandon the project.

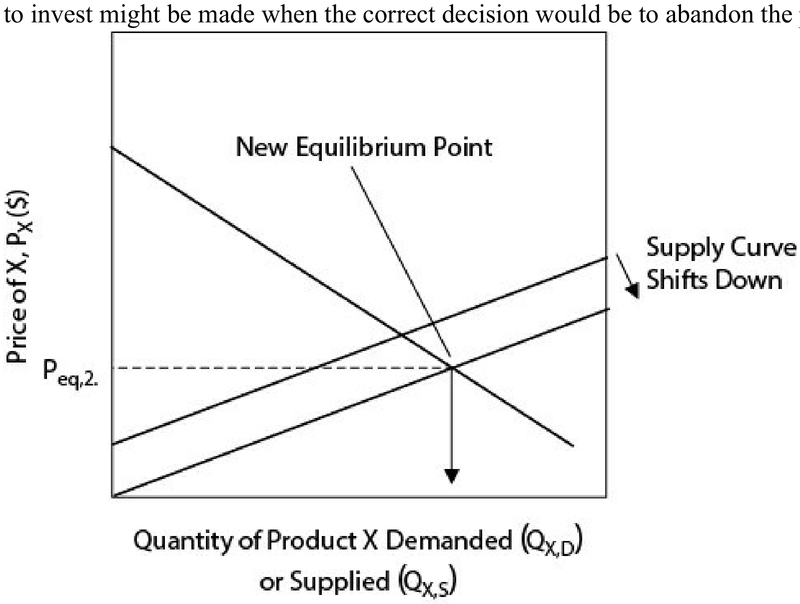


Figure 10.9 Illustration of Market Equilibrium for Product X When a New Plant Comes on Line

The situation is further complicated when competing products are considered. For example, if product Y can be used as a substitute for product X in some applications, then factors that affect Y will also affect X. It is easy to see that quantifying and predicting changes become very difficult. Torries [3] identifies important factors that affect both the shape and relative location of the supply and demand curves. These factors are listed in Table 10.2.

Table 10.2 Factors Affecting the Shape and Relative Location of the Supply and Demand Curves

Factors Influencing Supply	Factors Influencing Demand
Cost and amount of labor	Price of the product
Cost and amount of energy	Price of all substitute products

Cost and amount of raw material	Consumer disposable income
Cost of fixed capital (interest rates) and amount of fixed capital	Consumer tastes
Other miscellaneous factors	Manufacturing technology
	Other miscellaneous factors

(From Torries, T. F., *Evaluating Mineral Projects: Applications and Misconceptions*, by permission of SME, Littleton, CO, 1998; www.smenet.org)

In order to forecast accurately the prices of a product over a 10-or 15-year project, the factors in Table 10.2 need to be predicted. Clearly, even for the most well-known and stable products, this can be a daunting task. An alternative method to quantifying the individual supply and demand curves is to look at historical data for the product of interest.

The examination of historical data is a convenient way to obtain general trends in pricing. Such data represent the change in equilibrium price for a product with time. Often such data fluctuate widely, and although long-term trends may be apparent, predictions for the next one or two years will often be wildly inaccurate. For example, consider the data for average gasoline prices over the period January 1995 to June 2016, as shown in Figure 10.10. The straight line is a regression through the data and represents the best linear fit of the data. If this were the forecast for gasoline prices over this period, it would be a remarkably good prediction. However, even with this predicting line, significant variations in actual product selling price are noted. The maximum positive and negative deviations are +75¢/gal (+30%) and −45¢/gal (−24%). To illustrate further the effect of these deviations on profitability calculations, consider a new refinery starting production in late 2012. For this new plant, the selling price for its major product (gasoline) over the four-year period after startup drops by $1.45/gal. If this refinery were contracted to buy crude oil at a price fixed previously, then the profitability of the plant over this initial four-year period would be severely diminished and it would probably lose money.

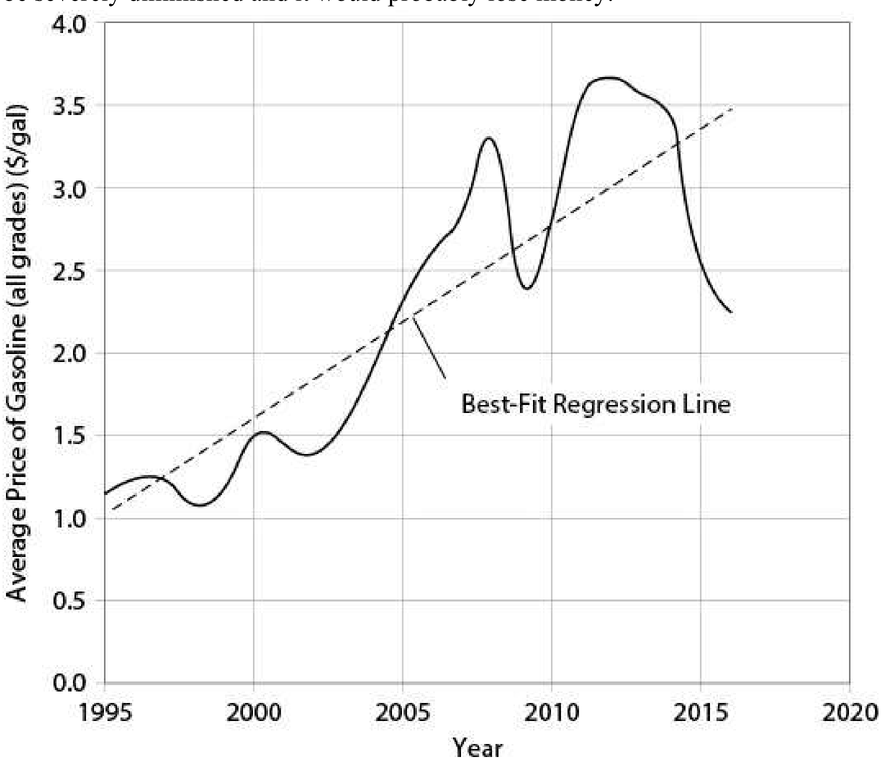


Figure 10.10 Average Price of All Grades of Gasoline over the Period January 1995 to June 2016 (from www.eia.doe.gov)

From the brief discussion given above, it is clear that predicting or forecasting future prices for chemical products is a very inexact and risky business. Perhaps it is best summed up by a quote attributed to "baseball philosopher" Yogi Berra [4]:

"It's tough to make predictions, especially about the future."

In the following section, it will be assumed that such predictions are available and will be used as given, known quantities. The question then is how much meaning can be placed on the results of these predictions when the input data—the basic variability of the parameters—is often poorly known. The answer is that by investigating and looking at how these parameters affect profitability, a better picture of how this variability or uncertainty affects the overall profitability of a project can be obtained. In general, this type of information is much more useful than the single-point estimate of profitability that has been considered up to this point.

10.7.2 Quantifying Risk

It should be noted that the quantification of risk in no way eliminates uncertainty. Rather, by quantifying it, a better feel can be developed for how a project's profitability may vary. Therefore, more informed and rational decisions regarding whether to build a new plant can be made. However, the ultimate decision to invest in a new chemical process always involves some element of risk.

Scenario Analysis. Returning to Example 10.1 regarding the profitability analysis for a new chemical plant, assume that, as the result of previous experience with similar chemicals and some forecasting of supply and demand for this new product, it is believed that the product price may vary in the range –20% to +5%, the capital investment may vary between –20% and +30%, and the cost of manufacturing may vary in the range –10% to +10%. How can these uncertainties be quantified?

One way to quantify uncertainty is via a **scenario analysis.** In this analysis, the best-and worst-case scenarios are considered and compared with the base case, which has already been calculated. The values for the three parameters for the two cases are given in Table 10.3.

Table 10.3 Values for Uncertain Parameters for the Scenario Analysis (All $ Figures in Millions)

Parameter	Worst Case	Best Case
Revenue, *R*	−20% = ($75)(0.8) = $60	+5% = ($75)(1.05) = $78.75
Cost of manufacture, *COMd*	+10% = ($30)(1.1) = $33	−10% = ($30)(0.9) = $27
Capital investment, *FCIL*	+30% = ($150)(1.3) = $195	−20% = ($150)(0.8) = $120

Next, these values are substituted into the spreadsheet shown in Table E10.1, and all the cash flows are discounted back to the start of the project to estimate the NPV. The results of these calculations are shown in Table 10.4.

Table 10.4 Net Present Values (NPVs) for the Scenario Analysis (All $ Figures in Millions)

Case	Net Present Value
Worst Case	−$59.64
Base Case	$17.12
Best Case	$53.62

The results in Table 10.4 show that, in the worst-case scenario, the NPV is very negative and the project will lose money. In the best-case scenario, the NPV is increased over the base case by approximately $35 million. From this result, the decision on whether to go ahead and build the plant is not obvious. On one hand, the process could be highly profitable, but on the other hand, it could lose nearly $60 million over the course of the ten-year plant life. By taking a very conservative philosophy, the results of the worst-case scenario suggest a decision of "do not invest." However, is the worst-case scenario realistic? Most likely, the worst-case (best-case) scenario is unduly pessimistic (optimistic). Consider each of the three parameters in Table 10.3. It will be assumed that the value of the parameter has an equal chance of being at the high, base-case, or low value. Therefore, in terms of probabilities, the chance of the parameter taking each of these values is 1/3, or 33.3%. Because there are three parameters (*R*, FCI_L, and COM_d), each of which can take one of three values (high, base case, low), there are $3^3 = 27$ combinations as shown in Table 10.5.

Table 10.5 Possible Combinations of Values for Three Parameters

Scenario	*R**	COM_d*	FCI_L*	Probability of Occurrence
1	−20%	−10%	−20%	(1/3)(1/3)(1/3) = 1/27
2	−20%	−10%	0%	
3	−20%	−10%	+30%	
4	−20%	0%	−20%	
5	−20%	0%	0%	
6	−20%	0%	+30%	
7	−20%	+10%	−20%	
8	−20%	+10%	0%	
9 (worst)	**−20%**	**+10%**	**+30%**	
10	0%	−10%	−20%	
11	0%	−10%	0%	
12	0%	−10%	+30%	
13	0%	0%	−20%	
14 (base)	**0%**	**0%**	**0%**	
15	0%	0%	+30%	
16	0%	+10%	−20%	
17	0%	+10%	0%	
18	0%	+10%	+30%	
19 (best)	**+5%**	**−10%**	**−20%**	
20	+5%	−10%	0%	
21	+5%	−10%	+30%	

22	+5%	0%	−20%	
23	+5%	0%	0%	
24	+5%	0%	+30%	
25	+5%	+10%	−20%	
26	+5%	+10%	0%	
27	+5%	+10%	+30%	(1/3)(1/3)(1/3) = 1/27

*0% refers to the base-case value.

From Table 10.5, it can be seen that Scenario 9 is the worst case and Scenario 19 is the best case. Either of these two cases has a 1 in 27 (or 3.7%) chance of occurring. Based on this result, it is not very likely that either of these scenarios would occur, so care should be taken in evaluating the scenario analysis. This is indeed one of the main shortcomings of the scenario analysis [2]. In reviewing Table 10.5, a better measure of the expected profitability might be the weighted average of all 27 possible outcomes. The idea of weighting results based on the likelihood of occurrence is the basis of the probabilistic approach to quantifying risk that will be discussed shortly. However, before looking at that method, it is instructive to determine the sensitivity of the profitability of the project to changes in important parameters. Sensitivity analysis is covered in the next section.

Sensitivity Analysis. To a great extent, the risk associated with the variability of a given parameter is dependent on the effect that a change in that parameter has on the profitability criterion of interest. For the sake of this discussion, the NPV will be used as the measure of profitability. However, this measure could just as easily be the DCFROR, DPEP, or any other profitability criterion discussed in Section 10.2. If it is assumed that the NPV is affected by n parameters ($x_1, x_2, x_3, \ldots, x_n$), then the first-order sensitivity to parameter x_1 is given in mathematical terms by the following quantity:

$$S_1 = \left[\frac{\partial(\text{NPV})}{\partial x_1}\right]_{x_2, x_3, \ldots, x_n} \qquad (10.6)$$

where the partial derivative is taken with respect to x_1, while holding all other parameters constant at their mean value. The sensitivity, S_1, is sometimes called a **sensitivity coefficient**. In general, this quantity is too complicated to obtain via analytical differentiation; hence, it is obtained by changing the parameter by a small amount and observing the subsequent change in the NPV, or

$$S_1 \approx \left[\frac{\Delta(\text{NPV})}{\Delta x_1}\right]_{x_2, x_3, \ldots, x_n} \qquad (10.7)$$

In Example 10.15, Example 10.1 is revisited to illustrate how the sensitivities of the revenue, cost of manufacturing, and fixed capital investment on the NPV are calculated.

Example 10.15

For the chemical process considered in Example 10.1, calculate the sensitivity of R, COM_d, and FCI_L and plot these sensitivities with respect to the NPV.

Solution

The effect of a 1% change is considered (½% on either side of the base case) in each parameter on the NPV. These results are shown in Table E10.15.

Table E10.15 Calculations for Sensitivity Analysis for Example 10.1 (All $ Figures Are in Millions)

Parameter	Value	NPV	Value	NPV	S_i
x_1 (Revenue, R)	+0.5% ($75.375/y)	$18.17	−0.5% ($74.625/y)	$16.07	$\frac{(18.17-16.07)}{(75.375-74.625)} = \frac{2.1}{0.75} = 2.80\text{y}$
x_2 (COM_d)	+0.5% ($30.150/y)	$16.70	−0.5% ($29.850/y)	$17.54	$\frac{(16.70-17.54)}{(30.15-29.85)} = \frac{-0.84}{0.30} = -2.80\text{y}$
x_3 (FCI_L)	+0.5% ($150.75)	$16.68	−0.5% ($149.25)	$17.56	$\frac{(16.68-17.56)}{(150.75-149.25)} = \frac{-0.88}{1.50} = -0.59\text{y}$

The fact that $S_1 = -S_2$ should not be surprising because, in the calculation of yearly cash flows, whenever R appears COM_d is subtracted from it (see Table E10.1). The changes in NPV for percent changes in each parameter are illustrated in Figure E10.15. The slopes of the lines are not equal to the sensitivities, because the x-axis is the percent change rather than the actual change in the parameter.

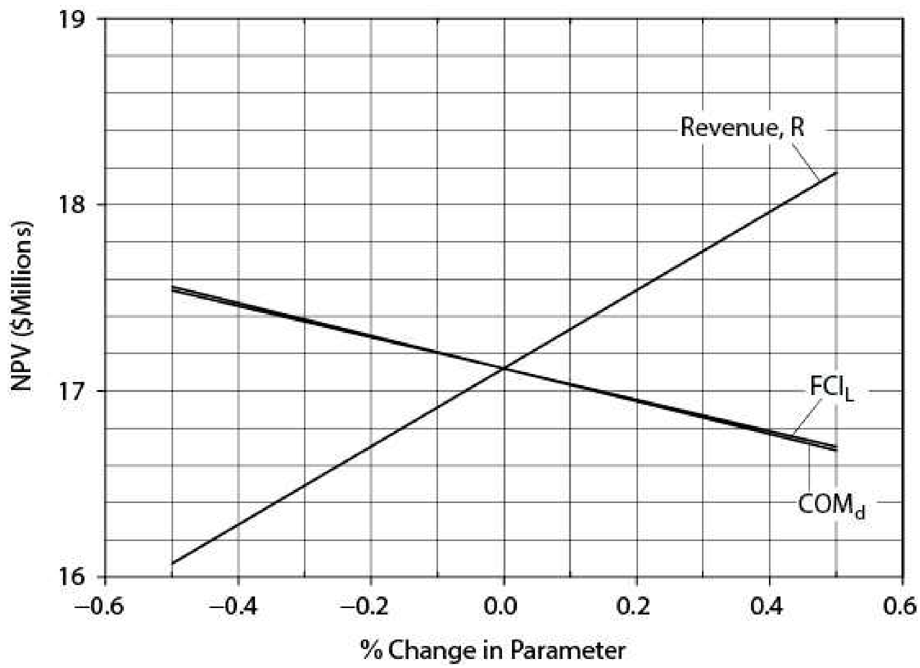


Figure E10.15 Sensitivity Curves for the Parameters Considered in Example 10.15

How can the sensitivity values calculated in Example 10.15 be used to estimate changes in the profitability criterion of the process? For small changes in the parameters, it may be assumed that the sensitivities are constant and can be added. Therefore, the change in NPV can be predicted for a set of changes in the parameters using the relationship in Equation (10.8):

$$\Delta \text{NPV} = S_1 \Delta x_1 + S_2 \Delta x_2 + \ldots + S_n \Delta x_n \qquad (10.8)$$

Example 10.16 illustrates this concept.

Example 10.16

What is the change in the NPV for a 2% increase in revenue coupled with a 3% increase in *FCI_L?*

Solution

Using Equation 10.8 and the results from Example 10.15 gives

$$\Delta NPV = S_1 \Delta x_1 + S_2 \Delta x_2 + S_3 \Delta x_3 = (2.80)(0.02)(75) - (2.80)(0) - (0.59)(0.03)(150) = \$1.545\text{million}$$

A Probabilistic Approach to Quantifying Risk: The Monte-Carlo Method. The basic approach adopted here will involve the following steps:

All parameters for which uncertainty is to be quantified are identified.

Probability distributions are assigned for all parameters in Step 1.

A random number is assigned for each parameter in Step 1.

Using the random number from Step 3, the value of the parameter is assigned using the probability distribution (from Step 2) for that parameter.

Once values have been assigned to all parameters, these values are used to calculate the profitability (NPV or other criterion) of the project.

Steps 3, 4, and 5 are repeated many times (for example, 1000).

A histogram and cumulative probability curve for the profitability criteria calculated from Step 6 are created.

The results of Step 7 are used to analyze the profitability of the project.

The algorithm described in this eight-step process is best illustrated by means of an example. However, before these steps can be completed, it is necessary to review some basic probability theory.

Probability, Probability Distribution, and Cumulative Distribution Functions. A detailed analysis and description of probability theory are beyond the scope of this book. Instead, some of the basic concepts and simple distributions are presented. The interested reader is referred to Resnick [5], Valle-Riestra [6], and Rose [7] for further coverage of this subject.

For any given parameter for which uncertainty exists (and to which some form of distribution will be assigned), the uncertainty must be described via a probability distribution. The simplest distribution to use is a uniform distribution, which is illustrated in Figure 10.11.

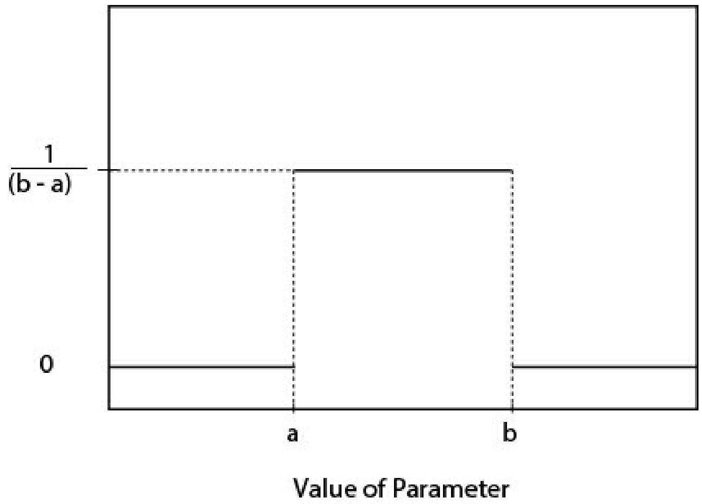


Figure 10.11 Uniform Probability Density Function

From Figure 10.11, the parameter of interest can take on any value between *a* and *b* with equal likelihood. Because the uniform distribution is a probability density function, the area under the curve must equal 1, and hence the value of the frequency (*y*-axis) is equal to 1/(*b*–*a*). The probability density function can be integrated to give the cumulative probability distribution, which for the uniform distribution is given in Figure 10.12.

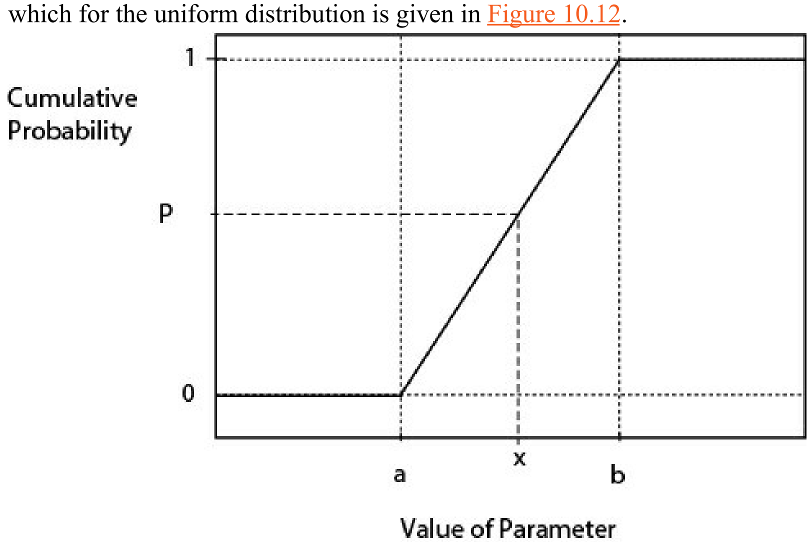


Figure 10.12 Cumulative Probability Distribution for a Uniform Probability Density Function

Figure 10.12 is interpreted by realizing that the probability of the parameter being less than or equal to *x* is *P*. Alternatively, a random, uniformly distributed value of the parameter can be assigned by choosing a random number in the range 0 to 1 (on the *y*-axis) and reading the corresponding value of the parameter, between *a* and *b*, on the *x*-axis. For example, if the random number chosen is *P*, then, using Figure 10.12, the corresponding value of the parameter is *x*. Clearly, the shapes of the density function and the corresponding cumulative distribution influence the values of the parameters that are used in the eight-step algorithm. Which probability density function should be used? Clearly, if frequency occurrence data for a given parameter are available, the distribution can be constructed. However, complete information about the way in which a given parameter will vary is often not available. The minimum data set would be the most likely value (*b*), and estimates of the highest (*c*) and lowest (*a*) values that the parameter could reasonably take. With this information, a triangular probability density function or distribution, shown in Figure 10.13, can be constructed. The corresponding cumulative distribution is shown in Figure 10.14. The equations describing these distributions are as follows:

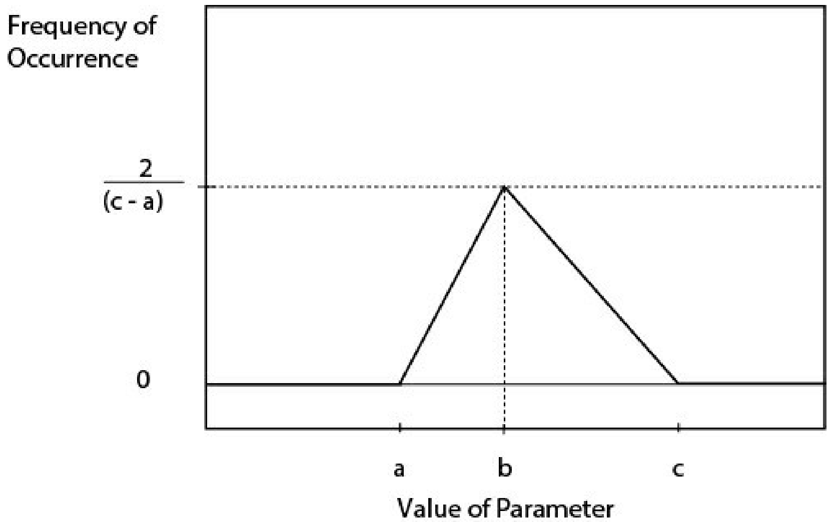


Figure 10.13 Probability Density Function for Triangular Distribution

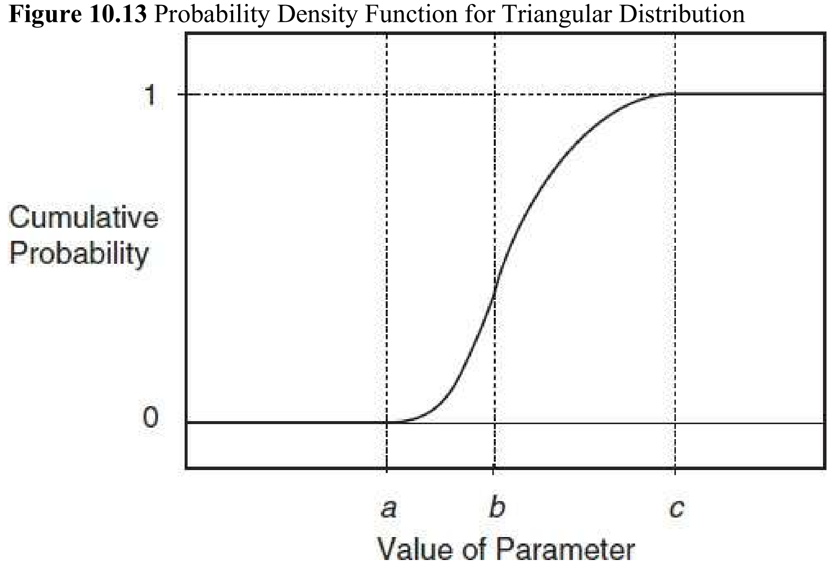


Figure 10.14 Cumulative Probability Function for Triangular Distribution

Triangular probability density function:

$$p(x) = \frac{2(x-a)}{(c-a)(b-a)} \text{for} x \leq b$$
$$p(x) = \frac{2(c-x)}{(c-a)(c-b)} \text{for} x > b \qquad (10.9)$$

Triangular cumulative probability function:

$$P(x) = \frac{(x-a)^2}{(c-a)(b-a)} \text{for} x \leq b$$
$$P(x) = \frac{(b-a)}{(c-a)} + \frac{(x-b)(2c-x-b)}{(c-a)(c-b)} \text{for} x > b \qquad (10.10)$$

Clearly, any probability density function and corresponding cumulative probability distribution could be used to describe the uncertainty in the data. Trapezoidal, normal, lognormal, and so on, are used routinely to describe uncertainty in data. However, for simplicity, the following discussions are confined to triangular distributions. The eight-step method for quantifying uncertainty in profitability analysis is illustrated next.

Monte-Carlo Simulation. The Monte-Carlo (M-C) method is simply the concept of assigning probability distributions to parameters, repeatedly choosing variables from these distributions, and using these values to calculate a function dependent on the variables. The resulting distribution of calculated values of the dependent function is the result of the M-C simulation. Therefore, the eight-step procedure is simply a specific case of the M-C method. Each of the eight steps is illustrated using the example discussed previously in the scenario analysis.

Step 1: All the parameters for which uncertainty is to be quantified are identified. Returning to Example 10.1, historical data suggest that there is uncertainty in the predictions for revenue (*R*), cost of manufacturing (COM_d), and fixed capital investment (FCI_L).

Step 2: Probability distributions are assigned for all parameters in Step 1. All the uncertainties associated with these parameters are assumed to follow triangular distributions with the properties given in Table 10.6.

Table 10.6 Data for Triangular Distributions for R, COM_d, and FCI_L (All $ Figures are in Millions)

Parameter	Minimum Value (a)	Most Likely Value (b)	Maximum Value (c)
Revenue, R	$60.0/y	$75.0/y	$78.75/y
Cost of manufacturing, COM_d	$27.0/y	$30.0/y	$33.0/y
Fixed capital investment, FCI_L	$120.0	$150.0	$195.0

Step 3: A random number is assigned for each parameter in Step 1. First, random numbers between 0 and 1 are chosen for each variable. The easiest way to generate random numbers is to use the Rand() function in Microsoft's Excel program or a similar spreadsheet. Tables of random numbers are also available in standard math handbooks [8].

Step 4: Using the random number from Step 3, the value of the parameter is assigned using the probability distribution (from Step 2) for that parameter. With the value of the random number equal to the right-hand side of Equation (10.10) and using the corresponding values of *a, b*, and *c*, this equation is solved for the value of *x*. The value of *x* is the value of the parameter to use in the next step. Table 10.7 illustrates this procedure for R, COM_d, and FCI_L.

Table 10.7 Illustration of the Assignment of Random Values to the Parameters R, COM_d, and FCI_L (All $ Figures Are in Millions)

Parameter	Random Number	Random Value of Parameter	NPV
Revenue (R)	0.3501	$69.92/y	
Cost of manufacturing (COM_d)	0.6498	$30.49/y	$−15.60
Fixed capital investment (FCI_L)	0.9257	$179.16	

To illustrate how the random values for the parameters are obtained, consider the calculation for COM_d. The number 0.6498 was chosen at random from a uniform distribution in the range 0–1 using Microsoft's Excel spreadsheet. This number, along with values of $a = 27$, $b = 30$, and $c = 33$, are then substituted for $P(x)$ in Equation (10.10) to give

$$0.6498 = \frac{(x-27)^2}{(33-27)(30-27)} = \frac{(x-27)^2}{18} \text{ for } x \le b$$

$$0.6498 = \frac{(30-27)}{(33-27)} + \frac{(x-30)(2(33)-x-30)}{(33-27)(33-30)} = 0.5 + \frac{(x-30)(36-x)}{18} \text{ for } x > b$$

From Equation (10.10), the $P(x)$ value for $x = b$ is given by

$$P(x=30) = \frac{(30-27)^2}{(33-27)(30-27)} = \frac{9}{18} = 0.5$$

Because the value of the random number (0.6498) is greater than 0.5, the form of the equation for $x > b$ must be used. Solving for *x* yields

$$0.6498 = 0.5 + \frac{(x-30)(36-x)}{18}$$

$$x^2 - 66x + 1082.6964 = 0$$

$$x = \frac{66 \pm \sqrt{(66^2-(4)(1)(1082.6964)}}{(2)(1)} = 33 \pm 2.51 = 30.49, or 35.51 (\text{impossible})$$

Step 5: Once values have been assigned to all parameters, these values are used to calculate the profitability (NPV or other criterion) of the project. The spreadsheet given in Table E10.1 was used to determine the NPV using the values given in Table 10.7. The NPV is also shown in Table 10.7.

Step 6: Steps 3, 4, and 5 are repeated many times (say, 1000). For the sake of illustration, Steps 3, 4, and 5 were repeated 20 times to yield 20 values of the NPV. These results are summarized in Table 10.8.

Step 7: A histogram or cumulative probability curve is created for the values of the profitability criterion calculated from Step 6. Using the data from Table 10.8, a cumulative probability curve is constructed. To do this, the data are ordered from lowest (−28.20) to highest (28.27), and the cumulative probability of the NPV being less than or equal to the value on the *x*-axis is plotted. The results are shown in Figure 10.15. The dashed line simply connects the 20 data points for this simulation. This line shows several bumps that are due to the small number of simulations. The solid line represents the data for 1000 simulations, and it can be seen that this curve is essentially smooth. The 1000-point simulation was carried out using the

CAPCOST software accompanying the text. The use of the software is addressed at the end of this section.

Table 10.8 Results of the 20-Point Monte-Carlo Simulation

Run	Rand (1)	R($/y)	Rand (2)	COM_d($/y)	Rand (3)	FCI_L($)	NPV($)
1	0.3501	69.92	0.6498	30.49	0.9257	179.16	−15.60
2	0.4063	70.69	0.7859	31.04	0.5531	156.16	−1.45
3	0.8232	75.22	0.3046	29.34	0.7073	163.57	11.59
4	0.9691	77.28	0.6164	30.37	0.8207	170.40	10.45
5	0.4418	71.15	0.2386	29.07	0.7273	164.66	−0.34
6	0.7170	74.20	0.9794	32.39	0.8313	171.14	−4.23
7	0.5626	72.58	0.8368	31.29	0.8891	175.65	−8.84
8	0.9854	77.74	0.1836	28.82	0.8136	169.92	16.34
9	0.8200	75.19	0.7440	30.85	0.5268	155.04	12.31
10	0.6319	73.33	0.1320	28.54	0.3863	149.48	16.84
11	0.1712	66.94	0.9465	32.02	0.0406	129.56	0.99
12	0.4966	71.82	0.3921	29.66	0.5993	158.23	4.34
13	0.2781	68.84	0.1474	28.63	0.7533	166.14	−5.76
14	0.2312	68.06	0.4187	29.75	0.5165	154.60	−4.27
15	0.5039	71.90	0.0042	27.28	0.5681	156.82	12.04
16	0.2184	67.84	0.8629	31.43	0.5107	154.36	−9.44
17	0.7971	74.97	0.3452	29.49	0.0789	133.32	28.27
18	0.2068	67.63	0.7975	31.09	0.9803	186.85	−28.20
19	0.8961	76.05	0.5548	30.17	0.1497	138.35	26.43
20	0.4201	70.87	0.2047	28.92	0.5713	156.96	4.50

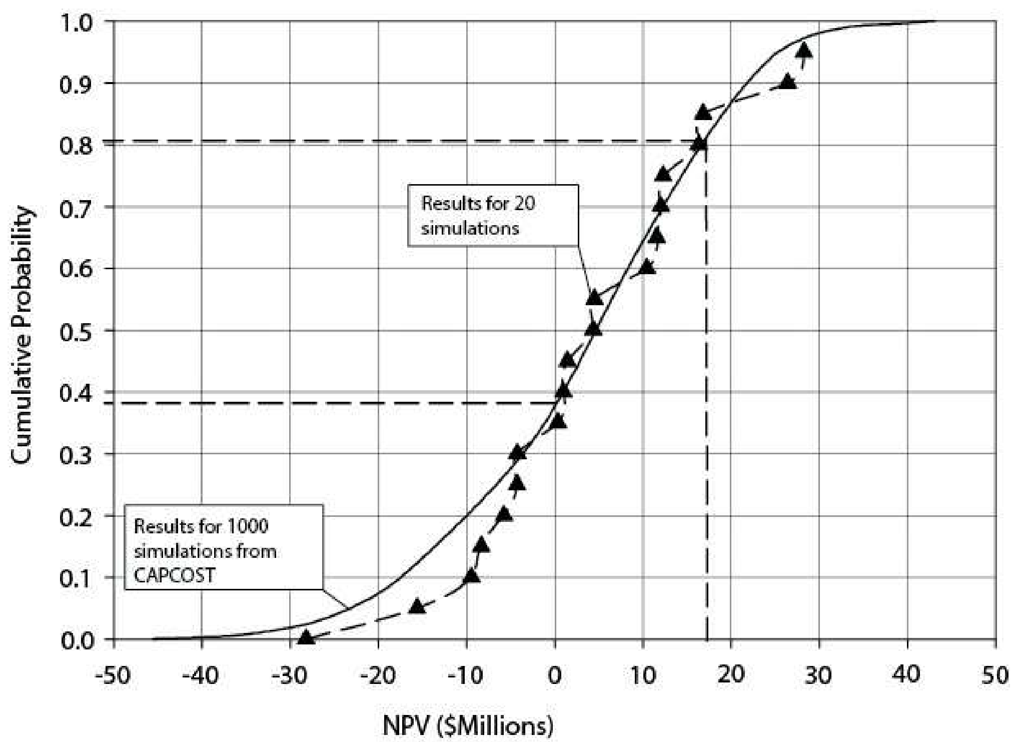


Figure 10.15 Cumulative Probability of NPV for Monte-Carlo Simulation

Step 8: The results of Step 7 are used to analyze the profitability of the project. From Figure 10.15, it can be seen that there is about a 38% chance that the project will not be profitable. The median NPV is about $5 million, and only about 21% of the values calculated lie above $17.12 million, which is the NPV calculated for the base case, using the most likely values of R, COM_d, and FCI_L.

Another way that the data from an M-C analysis can be used is in the comparison of alternatives. For example, consider two competing projects, A and B. A probabilistic analysis of both these projects yields the data shown in Figure 10.16. If only the median profitability is considered, corresponding to a cumulative probability of 0.5, then it might be concluded that Project A is better. Indeed, over a wide range of probabilities Alternative A gives a higher NPV than Alternative B. However, this type of comparison does not give the whole picture. By looking at the low end of NPV predictions, it is found that Project A has a 17% chance of returning a negative NPV compared with Project B, which is predicted to have only a 2% chance of giving a negative

NPV. Clearly, the choice regarding Projects A and B must be made taking into account both the probability of success and the magnitude of the profitability. The Monte-Carlo analysis allows a far more complete financial picture to be painted, and the decisions from such information will be more profound having taken more information into account.

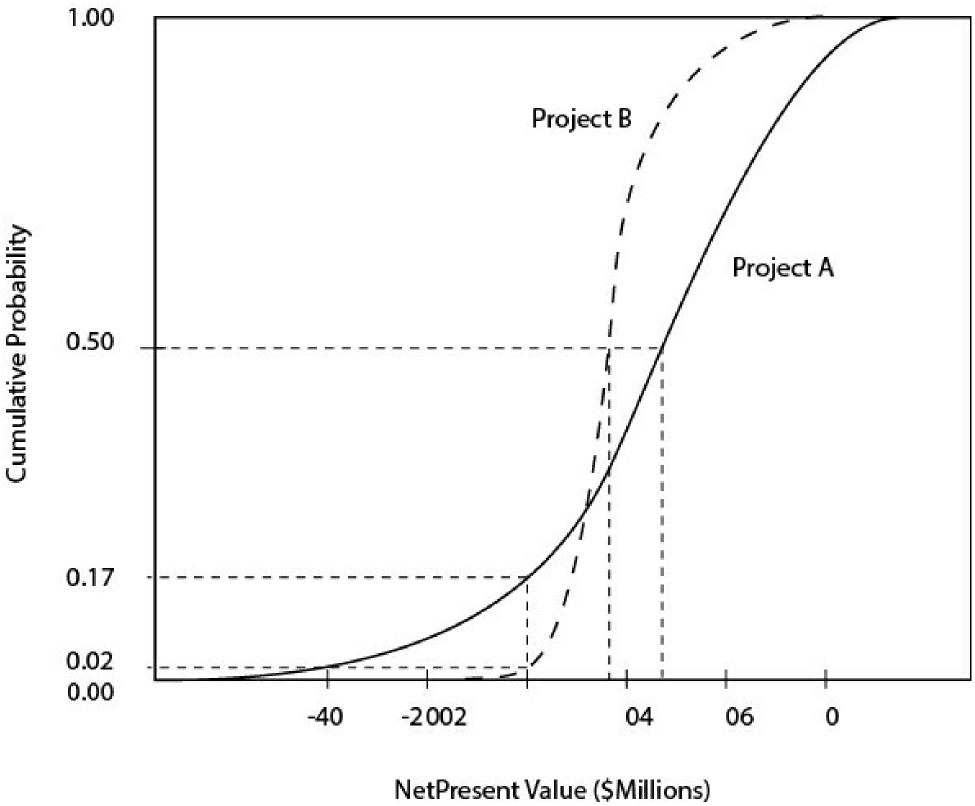


Figure 10.16 A Comparison of the Profitability of Two Projects Showing the NPV with Respect to the Estimated Cumulative Probability from a Monte-Carlo Analysis

Evaluation of the Risks Associated with Using New Technology. To this point, risks associated with predicting the items listed in Table 10.1 have been considered. For example, predictions for the variations associated with the cost of the plant, the cost of manufacturing, and the revenue generated by the plant were made. Then, by using the M-C technique, the relationship given in Figure 10.15 was generated. For processes using new technology, additional risks will be present, but these risks may be impossible to quantify in terms of the parameters given in Table 10.1. One way to take this additional risk into account is to assign a higher acceptable rate of return for projects using new technology compared with those using mature technologies. The effect of using a higher discount rate is to move the curve in Figure 10.15 to the left. This is illustrated in Figure 10.17. From this figure, it is apparent that if an acceptable rate of return is 15% p.a., then the project is not acceptable, whereas at a rate of 10% p.a. the project looks quite favorable. It can be argued that using a higher hurdle rate for new processes is unnecessary, because, for a new project, there will be greater ranges in the predictions of the variables, and this automatically makes the project using new technology "riskier," as the effect of broader ranges for variables is to flatten the NPV-probability curve. However, it may be impossible to estimate the effect of the new technology on, for example, the cost of manufacturing or the acceptance of a new product in the market. By specifying a higher acceptable rate of return on the investment for these projects, the interpretation between projects using new and old technologies is clear and unambiguous.

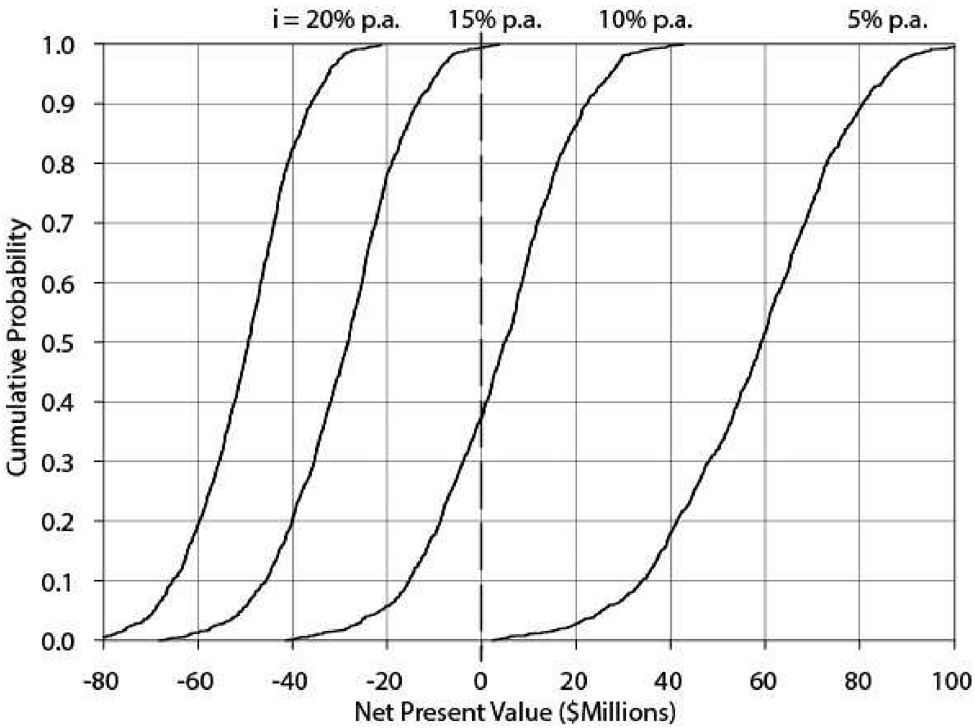


Figure 10.17 The Effect of Interest (Hurdle) Rate on Monte-Carlo Simulations

Monte-Carlo Analysis Using CAPCOST. The CAPCOST program introduced in Chapter 7 includes spreadsheets for estimating the cash flows of a project and the evaluation of profitability criteria such as NPV and DCFROR. In addition, a Monte-Carlo simulation has also been included that allows the following variables to be investigated:

FCI_L

Price of product

Working capital

Income tax rate

Interest rate

Raw material price

Salvage value

By specifying the ranges over which these terms are likely to vary, a Monte-Carlo analysis for a given problem can be achieved. Distributions of criteria such as NPV and DPBP are automatically given. The reader should consult the help file on the website for a tutorial on the use of this software.

10.8 PROFIT MARGIN ANALYSIS

All the techniques that have been discussed in this chapter use the fixed capital cost and the operating costs to evaluate the profitability of a process. Clearly, the accuracy of such predictions depends on the accuracy of the estimates for the different costs. When screening alternative processes, it is sometimes useful to evaluate the difference between the revenue from the sale of products and the cost of raw materials. This difference is called the **profit margin** or sometimes just the **margin**.

$$\textbf{Profit Margin} = \Sigma(\textbf{Revenue Products}) - \Sigma(\textbf{Cost of Raw Materials}) \quad (10.11)$$

If the profit margin is negative, the process will never be profitable. This is because no capital cost, utility costs, and other ancillary operating costs have been taken into account. A positive profit margin does not guarantee that the process will be profitable but does suggest that further investigation may be warranted. Therefore, the profit margin is a useful, but limited, tool for the initial screening of process alternatives. This is illustrated in Example 10.17.

Example 10.17

Consider the maleic anhydride process shown in Appendix B.5. Estimate the profit margin for this process using the costs of raw materials and products from Table 8.4.

Solution

From Tables 8.4 and B.5.1 the following flowrates and costs are found: Cost of benzene = $1.196/kg

Cost of maleic anhydride = $1.543/kg

Feed rate of benzene to process (Stream 1, Figure B.5.1) = 3304 kg/h

Product rate of maleic anhydride (Stream 13, Figure B.5.1) = (24.8)(98.058) = 2432 kg/h

Profit Margin = (2432)(1.543) − (3304)(1.196) = − $199.01/h or − $199.01/(2432) = − $0.082/kg of maleic anhydride

Clearly, from an analysis of the profit margin, further investigation of the maleic anhydride process is not warranted.

10.9 SUMMARY

In this chapter, the basics of profitability analysis for projects involving large capital expenditures were covered. The concepts of nondiscounted and discounted profitability criteria were introduced, as were the three bases for these criteria: time, money, and interest rate.

How to choose the economically optimum piece of equipment among a group of alternatives using the capitalized cost, the equivalent annual operating cost, and the common denominator methods was demonstrated.

The concept of incremental economic analysis was introduced and applied to an example involving large capital budgets and also to a retrofit project. It was shown that both the net present value (NPV) and the equivalent annual operating cost (EAOC) methods were particularly useful when comparing alternatives using discounted cash flows.

Finally, the concept of assigning probabilities to variables in order to quantify risk was discussed. The Monte-Carlo technique was introduced, and its application to simulate the cumulative distribution of net present values of a project was described. The interpretation of results from this technique was presented. Finally, the simulation of risk and the analysis of data using the CAPCOST package was illustrated by an example.

WHAT YOU SHOULD HAVE LEARNED

There are discounted (including the time value of money) and nondiscounted methods (which do not include the time value of money) for estimating profitability, which may give different results.

There are time-based, cash-based, and interest-rate-based methods for estimating profitability, which usually give the same result within the same discounted or nondiscounted category.

Incremental analysis is needed when comparing alternatives that require different amounts of capital expenditures.

NPV, EAOC, and DCFROR tend to give the most reliable results.

Monte-Carlo analysis can be used to include the effect of parameter uncertainty in the NPV, EAOC, and DCFROR.

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SHORT ANSWER QUESTIONS

1. The evaluation of a project requiring a large capital investment has yielded an NPV (net present value) of $\$20 \times 10^6$. If the internal hurdle rate for this project was set at 10% p.a., will the DCFROR (discounted cash flow rate of return) be greater or less than 10%? Explain.

2. The following are results of a recent evaluation of two projects. Which would you choose? Defend your choice. Your opportunity cost for capital is 15%.

	NPV	DCFROR
Project A	10	55%
Project B	10,000	16%

3. Explain the concept of an incremental economic analysis.

4. When comparing two pieces of equipment for a given service, if each piece of equipment has the same life and each costs the same, would the amount of maintenance required for the equipment be an important factor? Why?

5. What economic criterion would you use to choose the best piece of equipment among three alternatives, each with a different operating cost, capital cost, and equipment life?

6. Do you agree with the statement "Monte-Carlo simulation enables the design engineer to eliminate risk in economic analysis"? Please explain your answer.

7. In evaluating a large project, what are the advantages and disadvantages of probabilistic analysis?

PROBLEMS

For the following problems, unless stated otherwise, you may assume that the cost of land, *L*, and the salvage value, *S*, of the plant are both zero.

8. The projected costs for a new plant are given below (all numbers are in $\$10^6$).

Land cost = $7.5

Fixed capital investment = $120 ($60 at end of year 1, $39.60 at end of year 2, and $20.40 at end of year 3)

Working capital = $35 (at startup)
Startup at end of year 3
Revenue from sales = $52
Cost of manufacturing (without depreciation) = $18
Tax rate = 40%
Depreciation method = MACRS over 5 years
Length of time over which profitability is to be assessed = 10 years after startup
Internal rate of return = 9.5% p.a.
For this project, do the following:
Draw a cumulative (nondiscounted) after-tax cash flow diagram.
From Part (a), calculate the following nondiscounted profitability criteria for the project:
Cumulative cash position and cumulative cash ratio
Payback period
Rate of return on investment
Draw a cumulative (discounted) after-tax cash flow diagram.
From Part (c), calculate the following discounted profitability criteria for the project:
Net present value and net present value ratio
Discounted payback period
Discounted cash flow rate of return (DCFROR)
9. Repeat Problem 10.8 using a straight-line depreciation method over 7 years. Compare the results with those obtained in Problem 10.8. Which depreciation method would you use?
10. The following expenses and revenues have been estimated for a new project:
Revenues from sales = $4.1 × 10^6/y
Cost of manufacturing (excluding depreciation) = $1.9 × 10^6/y
Taxation rate = 40%
Fixed capital investment = $7.7 × 10^6
(two payments of $5 × 10^6 and $2.7 × 10^6 at the end of years 1 and 2, respectively)
Startup at the end of year 2
Working capital = $2 × 10^6 at the end of year 2
Land cost = $0.8 × 10^6 at the beginning of the project (time = 0)
Project life (for economic evaluation) = 10 y after startup
For this project, estimate the NPV of the project assuming an after-tax internal hurdle rate of 11% p.a., using the following depreciation schedules:
MACRS method for 5 years
Straight-line depreciation with an equipment life (for depreciation) of 9.5 years
Comment on the effect of discounting on the overall profitability of large capital projects.
11. In reviewing current operating processes, the company accountant has provided you with the following information about a small chemical process that was built ten years ago.
Capital investment = $30 × 10^6 ($10 × 10^6 at the end of year 1, $15 × 10^6 at the end of year 2, $5 × 10^6 at the end of year 3.
Working capital = $10 × 10^6)

Year after Startup	**Yearly After-Tax Cash Flow ($$10^6$/y)**
1	7.015
2	6.206
3	6.295
4	6.852
5	6.859
6	7.218
7	5.954
8	5.459
9	5.789
10	5.898

Over the last ten years, the average (after-tax) return on investment that nonprocess projects have yielded is 10%.
What is the DCFROR for this project over the last 12 years? (Ignore land and working capital costs.)
In retrospect, was the decision to build this plant a good one?

12. The after-tax cash flows for a new chemical process are shown in Table P10.12. Using these data, calculate the following:
Payback period (PBP)
Cumulative cash position (CCP) and cumulative cash ratio (CCR)
Rate of return on investment (ROROI)
Discounted payback period (DPBP)
Net present value (NPV)
Discounted cash flow rate of return (DCFROR)

Table P10.12 Nondiscounted Cash Flow Calculations for Problem 10.12 (All Figures Are in $Millions)

End of Year	Capital Investment	Depreciation Allowance	Revenue from Sales	Total Annual Costs	Net Profit	Income Tax	Net Profit after Tax	After-Tax Cash Flow
0	(10)*, **	—	—	—	—	—	—	(10)
1	(25)	—	—	—	—	—	—	(25)
2	(20)	—	—	—	—	—	—	(20)
3	(15 + 20)†, ‡	—	—	—	—	—	—	(35)
4		8.57	60	50	10	0.54	0.89	9.46
5	—	8.57	120	92	28	7.38	12.05	20.62
6	—	8.57	120	47	73	24.48	39.95	48.52
7	—	8.57	120	50	70	23.34	38.09	46.66
8	—	8.57	120	60	60	19.54	31.89	40.46
9	—	8.57	120	51	69	22.96	37.47	46.04
10	—	8.57	120	40	80	27.14	44.29	52.86
11	—	—	120	40	80	30.40	49.60	49.60
12		—	120	40	80	30.40	49.60	49.60
13	30	—	120	40	80	30.40	49.60	79.60

*Numbers in () represent negative values.
**Land cost = 10.
†Plant started up at end of year 3.
‡Working capital = 15.
Use a 10% discount rate for Parts (d) and (e).

13. From the data given in Table P10.12, determine the following information regarding the calculations performed for this analysis:
What taxation rate was assumed?
What was the total fixed capital investment?
What method of depreciation was used?
What was the cost of manufacturing, not including depreciation?

14. Consider the following two new chemical plants, each with an initial fixed capital investment (year 0) of $\$15 \times 10^6$. Their cash flows are as follows:

Year	Process 1 ($million/y)	Process 2 ($million/y)
1	3.0	5.0
2	8.0	5.0
3	7.0	5.0
4	5.0	5.0
5	2.0	5.0

Calculate the NPV of both plants for interest rates of 6% and 18%. Which plant do you recommend? Explain your results.
Calculate the DCFROR for each plant. Which plant do you recommend?
Calculate the nondiscounted payback period (PBP) for each plant. Which plant do you recommend?
Explain any differences in your answers to Parts (a), (b), and (c).

15. In a design, you have the choice of purchasing one of the following batch reactors:

Costs	A	B	C
Material of Construction	CS	SS	Hastalloy
Installed Cost	$15,000	$25,000	$40,000
Equipment Life	3 y	5 y	7 y
Yearly Maintenance Cost	$4000	$3000	$2000

If the internal rate of return for such comparisons is 9% p.a., which of the alternatives is least costly?

16. Two pieces of equipment are being considered for an identical service. The installed costs and yearly operating costs associated with each piece of equipment are as follows:

Costs	**A**	**B**
Installed Cost	$5000	$10,000
Operating Cost	$2000/y	$1000/y
Equipment Life	5 y	7 y

If the internal hurdle rate for comparison of alternatives is set at 15% p.a., which piece of equipment do you recommend be purchased?

Above what internal hurdle rate would you recommend Project A?

17. Your company is considering modifying an existing distillation column. A new reboiler and condenser will be required, along with several other peripherals. The equipment lists are as follows:

Option 1

Equipment	**Installed Cost ($)**	**Operating Cost ($/y)**	**Equipment Life (y)**
Condenser	50,000	7000	10
Reboiler	75,000	5000	15
Reflux Pump	7500	8000	10
Reflux Drum	12,500	—	10
Piping	8000	—	15
Valves	6500	—	10

Option 2

Equipment	**Installed Cost ($)**	**Operating Cost ($/y)**	**Equipment Life (y)**
Condenser	75,000	4000	15
Reboiler	75,000	5000	15
Reflux Pump	10,500	5000	15
Reflux Drum	14,500	—	15
Piping	8000	—	15
Valves	6500	—	10

The internal hurdle rate for comparison of investments is set at 12% p.a. Which option do you recommend?

18. Because of corrosion, the feed pump to a batch reactor must be replaced every three years. What is the capitalized cost of the pump?

Data:

Purchased cost of pump = $35,000

Installation cost = 75% of purchased cost

Internal hurdle rate = 10% p.a.

19. Three alternative pieces of equipment are being considered for solids separation from a liquid slurry:

Equipment type	**Capital Investment ($)**	**Operating Cost ($/y)**	**Service Life (y)**
Rotary Vacuum Filter	15,000	3000	6
Filter Press	10,000	5000	8
Hydrocyclone and Centrifuge	25,000	2000	10

If the internal hurdle rate for this project is 11% p.a., which alternative do you recommend?

20. A change in air pollution control equipment is being considered. A baghouse filter is being considered to replace an existing electrostatic precipitator. Consider the costs and savings for the project in the following data:

Cost of new baghouse filter = $250,000

Projected utility savings = $70,000/y

Time over which cost comparison should be made = 7 years

Internal hurdle rate = 7% p.a.

Should the baghouse filter be purchased and installed?

21. In considering investments in large capital projects, a company is deciding in which of the following projects it will invest:

(All Values in $million)	**Project A**	**Project B**	**Project C**
Capital Required (in year 0)	80	100	120
After-tax, yearly cash flow (years 1–10)	11	14	16

The company can always invest its money in stocks that are expected to yield 5.5% p.a. (after tax). In which, if any, of the projects should the company invest if the capital ceiling for investment is $250 million and a project life of 10 years is assumed? Would you argue to raise the investment ceiling?

22. Because you live in a southern, warm-weather climate, your electricity bill is very large for about eight months of the year due to the need for air conditioning. You have been approached by an agency that would like to assist you in installing solar panels to provide electricity to run your air conditioning. You were also considering adding additional insulation to your house. You also have the option of doing nothing. Which of the following options would you choose based on the given data?
Do nothing.
Install only the solar collector.
Install only the insulation.
Install both the solar collector and the insulation.
Data (all figures in $thousands)

Purchase and installation cost of solar collector	25
Purchase and installation of insulation	5
Current cooling bill	2.5/y
Expected savings from insulation alone	0.9/y
Expected savings from solar collector alone	2.0/y
Expected savings from insulation and solar collector	2.5/y
Other maintenance on house	2.0/y
Assessed value of house (2007)	300
Interest rate of savings if do not spend money	6.5% p.a.
Number of years assumed lifetime of insulation and solar collector	15

If there were a tax credit for installing the solar collector in the year in which it was installed, how much of a tax credit (in % of initial investment) would be required to make the solar collector alone a worthwhile investment?
23. You have been asked to evaluate several investment opportunities for the biotechnology company for which you work. These potential investments concern a new process to manufacture a new, genetically engineered pharmaceutical. The financial information on the process alternatives are as follows:

Case	**Capital Investment ($million)**	**Yearly, After-Tax Cash Flow ($million/y)**
Base	75	19
Alternative 1	15	3
Alternative 2	25	5
Alternative 3	30	7

For the alternatives, the capital investment and the yearly after-tax cash flows are incremental to the base case. The assumed plant life is 12 years, and all of the capital investment occurs at time = 0.
If an acceptable, nondiscounted rate of return on investment (ROROI) is 25% p.a., which is the best option?
If an acceptable, after-tax, discounted rate of return is 15% p.a., which option is the best?
24. Your company is considering investing in a process improvement that would require an initial capital investment of $500,000. The projected increases in revenue over the next seven years are as follows:

Year	**Incremental Revenue ($thousand/y)**
1	100
2	90
3	90
4	85
5	80
6	80
7	75

The company can always leave the capital investment in the stock portfolio, which is projected to yield 8% p.a. over the next seven years. What should the company do?
What is the break-even rate of return between the process improvement and doing nothing?
If the capital investment could be changed without changing the incremental revenues, what capital investment changes this investment from being profitable (not profitable) to being not profitable (profitable)?
25. During the design of a new process to manufacture nanocomposites, several alternative waste treatment processes are being considered. The base case is the main waste treatment process, and the options are modifications to the base case. Not all options are compatible with each other, and the economic data on the only possible combinations are as follows:

Case	**Capital Investment ($million)**	**Annual, After-Tax Cash Flow ($thousand/y)**
Base	5.0	750
Base + option 1	5.1	770

Base + option 1 + option 2	5.3	790
Base + option 3	5.2	782
Base + option 1 + option 3	5.4	810

The nondiscounted, internal hurdle rate for investment is 14% p.a., after tax. Which waste treatment process do you recommend?

26. The installation of a new heat exchanger is proposed for the batch crystallization step in an existing pharmaceutical manufacturing process. The heat exchanger costs $49,600 (installed) and saves $8000/y in operating costs. Is this a good investment based on a before-tax analysis?

Data:

Internal hurdle rate = 12% p.a. (after tax)

Taxation rate = 40%

5-year MACRS depreciation used

Service life of heat exchanger = 12 y

27. A process for the fabrication of microelectronic components has been designed. The required before-tax return on investment is 18% p.a., and the equipment life is assumed to be eight years.

	Base-Case Design	**Alternative 1**	**Alternative 2**
Capital Investment ($million)	21	—	—
Additional Investment ($million)	—	2.15	1.35
Product Revenue ($million/y)	12	12	12
Raw Material Costs ($million/y)	5.1	5.1	5.1
Other Operating Costs ($million/y)	0.35	0.27	0.31

Do you recommend construction of the base-case process?

Do you recommend including either of the process alternatives?

Suppose there were another alternative, compatible with either of the previous alternatives, requiring an additional $3.26 million capital investment. What savings in operating cost would be required to make this alternative economically attractive?

28. A new pharmaceutical plant is expected to cost (FCI_L) $25 million, and the revenue (R) from the sale of products is expected to be $10 million/y for the first four years of operation and $15 million/y thereafter. The cost of manufacturing without depreciation (COM_d) is projected to be $4 million/y for the first four years and $6 million/y thereafter. The cost of land (at the end of year zero) is $3 million, the working capital at startup (which occurs at the end of year 2) is $3 million, and the fixed capital investment is assumed to be paid out as $15 million at the end of year 1 and $10 million at the end of year 2. Yearly income starts in year 3, and the plant life is ten years after startup. The before-tax criterion for profitability is 17%. Assume the plant has no salvage value, but that the cost of land and the working capital are recovered at the end of the project life.

Draw a labeled, discrete, nondiscounted cash flow diagram for this project.

Draw a labeled, cumulative, discounted (to year zero) cash flow diagram for this project.

What is the NPV for this project? Do you recommend construction of this plant?

29. How much would you need to save annually to be willing to invest $1.75 million in a process improvement? The internal hurdle rate for process improvements is 18% before taxes over eight years.

30. How much would you be willing to invest in a process improvement to save $2.25 million/y? The internal hurdle rate for process improvements is 15% before taxes over six years.

31. You can invest $5.1 million in a process improvement to save $0.9 million/y. What internal hurdle rate would make this an attractive option? The assumed lifetime for these improvements is nine years.

32. Recommend whether your company should invest in the following process: The internal hurdle rate is 17% p.a., before taxes, over an operating lifetime of 12 years. The fixed capital investment is made in two installments: $5 million at time zero and $3 million at the end of year 1. At the end of year 1, $1 million in working capital is required. For the remainder of the project lifetime, $2 million in income is realized.

33. You are considering two possible modifications to an existing microelectronics facility. The criterion for profitability is 18% p.a. over six years. All values are in $million.

	Alternative 1	**Alternative 2**
Project Cost	2.25	3.45
Yearly Savings	0.65	0.75

What do you recommend based on a nondiscounted ROROII analysis?

What do you recommend based on an INPV analysis?

Based on the results of Parts (a) and (b), what do you recommend?

34. A new biotech plant is expected to cost (FCI_L) $20 million, with $10 million paid at the beginning of the project and $10

million paid at the end of year 1. There is no land cost because the land is already owned. The annual profit, before taxes, is $4 million and the working capital at startup (which occurs at the end of year 1) is $1 million. The plant life is 10 years after startup. The before-tax criterion for profitability is 12%. Assume that the plant has no salvage value, and that the working capital is recovered at the end of the project life.

Draw a labeled, discrete, nondiscounted cash flow diagram for this project.

Draw a labeled, cumulative, discounted (to year zero) cash flow diagram for this project.

What is the NPV for this project? Do you recommend construction of this plant?

What would the annual profit, before taxes, have to be for the NPV to be $2 million?

The plant is built and has been operational for several years. It has been suggested that $3 million be spent on plant modifications that will save money. Your job is to analyze the suggestion. The criterion for plant modifications is a 16% before-tax return over five years. How much annual savings are required before you would recommend in favor of investing in the modification?

35. You are responsible for equipment selection for a new micro-fiber process. A batch blending tank is required for corrosive service. You are considering three alternatives:

Alternative	Material	Cost ($thousand)	Maintenance Cost ($thousand/y)	Equipment Life(y)
A	Carbon Steel	5	5.5	3
B	Ni Alloy	15	3.5	5
C	Hastalloy	23	2.5	9

The after-tax internal hurdle rate is 14% p.a. Which alternative do you recommend?

36. In an already operational pharmaceutical plant, you are considering three alternative solvent-recovery systems to replace an existing combustion process. The internal hurdle rate is 15% after taxes over eight years. Which alternative do you recommend?

Alternative	Total Module Cost of System ($ thousands)	Natural Gas Savings ($ thousand/y)	Savings in Solvent Cost ($ thousand/y)	Net Maintenance Savings ($ thousand/y)
A	88	15	25	33
B	125	15	45	25
C	250	15	75	18

37. A condenser using refrigerated water is being considered for the recovery of a solvent used in a pharmaceutical coating operation. The amount of solvent recovered from the gas stream is a function of the temperature to which it is cooled. Three cases, each using a different exchanger and a different amount of refrigerated water, are to be considered. Data for the process cases are given in the table.

Case	Installed Cost of Exchanger ($)	Cost of Refrigerated Water ($/y)	Value of Acetone Recovered ($/y)
A	103,700	15,000	32,500
B	162,400	30,000	65,000
C	216,100	34,500	74,750

If the nondiscounted hurdle rate for projects is set at 15% p.a., which case do you recommend?

If the discounted hurdle rate is set at 5% p.a. and the project life is set at 7 years, which case do you recommend?

How does your result from Part (b) change if the project life were changed to 15 years?

How does your result from Part (b) change if the project life were changed to 5 years?

38. For Problem 10.8, uncertainties associated with predicting the revenues and cost of manufacturing are estimated to be as follows: **Revenue**: Expected range of variation from base case, low = $45 million, high = $53.5 million

COM$_d$: Expected range of variation from base case, low = $16 million, high = $21.5 million Using the above information, evaluate the expected distribution of NPVs and DCFRORs for the project. Would this analysis change your decision compared to that for the base case?

39. Calculate the lowest and highest NPVs that are possible for Problem 10.38. Compare these values with the distribution of NPVs from Problem 10.38. What are the probabilities of getting an NPV within $5 million of these values?

40. Perform a Monte-Carlo analysis on Problem 10.10, using the following ranges for uncertain variables (all figures in $millions):

	Low	High
FCI_L	6.6	9.3
Revenues	3.5	4.5
Interest rate (%)	9.5	12.0
COM_d	1.7	2.5

What do you conclude? *Hint*: When using the CAPCOST program, set the equation for $COM_d = C_{raw\ materials}$ and input the variability in COM_d as a variability in the cost of raw materials.

41. You are considering buying a house with a mortgage of $250,000. The current interest rates for a mortgage loan for a 15-year period are 7.5% p.a. fixed or 6.75% p.a. variable. Based on historical data, the variation in the variable rate is thought to be from a low of 6.0% to a high of 8.5%, with the most likely value as 7.25%. The variable interest rate is fixed at the beginning of each year. Answer the following questions:

What are the maximum and minimum yearly payments that would be expected if the variable interest option is chosen? For simplicity assume that the compounding period is one year.

Set up a Monte-Carlo simulation by picking 20 random numbers and using these to choose the variable interest rate for each of the 20 years of the loan.

Calculate the yearly payments for each year using the data from Part (b).

Hint: You should keep track of the interest paid on the loan and the remaining principal. The remaining principal is used to calculate the new yearly payment with the new yearly interest rate.

42. Perform a Monte-Carlo simulation on Example 10.1 for the following conditions. Show that the variation in the NPV is the same as shown in Figure 10.15. To which variable is the NPV more sensitive?

	Low	High
FCI_L	–20%	35%
Product Price	–25%	10%
Raw Material Price	–10%	25%

Note: Because the Monte-Carlo method is based on the generation of random numbers, no two simulations will be exactly the same. Therefore, you may see some small differences between your results and those shown in Figure 10.15.

43. A product is to be produced in a batch process. The estimated fixed capital investment is $5 million. The estimated raw materials cost is $100,000/batch, the estimated utility costs are $60,000/batch, and the estimated waste treatment cost is $23,000/batch. The revenue is predicted to be $220,000/batch. An initial scheduling scenario suggests that it will be possible to produce 22 batches/y. The internal hurdle rate is 15% p.a. before taxes over 10 years.

Is this process profitable?

What is the minimum number of batches/y that would have to be produced to make the process profitable?

If 26 batches/y is the maximum that can be produced, what is the rate of return on the process?

44. Three different products are manufactured in an existing batch process. The details are as follows:

Product	kg/batch	Product Value ($/kg)	Batches/y
A	5000	7.5	20
B	2500	6.25	13
C	4000	5.75	16

The cost of manufacturing is $0.95 million/y. The demand for these products is increasing, and the crystallization step has been determined to be the bottleneck to increasing the capacity. It is desired to add 25% capacity to this process. The internal hurdle rate for process improvements is 17% p.a. over five years.

If a new batch crystallizer, which allows for a 25% capacity increase, costs $750,000, do you recommend this process improvement?

Capital funds are tight, and it has been determined that the maximum investment possible is $600,000, resulting in a smaller new crystallizer. Using this crystallizer, identical profitability as found in Part (a) has been determined. Determine what capacity increase results from purchasing the smaller crystallizer.

Suppose that it is now possible to purchase the $750,000 crystallizer, thereby increasing capacity by 25%. However, the purchase of this crystallizer requires that the DCFROR (for this incremental investment) be at least 40% over five years. Is this DCFROR reached?

45. A batch process runs on a zero-wait-time schedule (see Chapter 3). It has been determined that a 20% increase in capacity is possible if three equally sized storage tanks are purchased, and the processing schedule is altered appropriately. The cost of manufacture is $1.5 million/y with revenues of $2.75 million/y. The internal hurdle rate for process improvements is 20% p.a. over six years.

If each tank costs $1.7 million, do you recommend the investment?

What is the maximum cost per storage tank that will meet the profitability criterion and result in a 20% increase in capacity?

Suppose that storage tanks each cost $2.0 million but still result in a 20% increase in capacity. What is the DCFROR for the recommended process improvement?

46. You are designing a new pharmaceutical facility. The criterion for profitability is 14% before taxes over ten years. The alternatives need not be included in the base design, but either one or both may be included if they are profitable. The values listed under the alternatives are in addition to the base case. All values are in $million.

	Base	Alternative 1	Alternative 2

Project Cost 25	5	3
Yearly Profit5	0.75	0.5

Do you recommend the base case? Use what you consider to be the most appropriate method and explain your justification for using it.

Do you recommend either or both of the alternatives based on a nondiscounted analysis?

Do you recommend either of both of the alternatives based on a discounted analysis?

47. The addition of a heat recovery system to a process is being considered. The costs of the heat exchangers and other economic parameters are: Installed cost of new heat exchangers = $4,500,000

Time over which cost comparison should be made = 8 years

Internal hurdle rate = 6.5% p.a.

What is the minimum yearly savings required for the heat recovery system to be economically attractive?

If the yearly savings were $1,050,000/y what would be the discounted payback period for the investment?

48. A plant that operates at high temperatures and moderate pressures is being constructed. You have been asked to evaluate two alternatives for the gasket materials used in the plant. These gasket materials have different lives, different replacement costs, and different maintenance costs: Gasket A

Life = 2 yr

Maintenance cost = $5/year/gasket

Replacement cost = $100/gasket

Gasket B

Life = 7 yr

Maintenance cost = $2/year/gasket

Replacement cost = $?/gasket

If the hurdle rate for such comparisons is 5% p.a., determine what the cost of Gasket B would be so that there is no advantage between the two gaskets.

49. You have been tasked with recommending a pump for a corrosive service by considering three manufacturers. The after-tax, internal hurdle rate is 10%. Which of the following three alternatives do you recommend and why?

Pump Manufacturer	Cost ($thousand)	Maintenance Cost ($thousand/y)	Equipment Life (y)
A	5	5	3
B	25	3	8
C	15	4	5

50. A pressure vessel needs to be purchased by evaluating three different options. The after-tax, internal hurdle rate is 10%. Which of the following three alternatives do you recommend and why?

Vessel Type	Cost ($thousand)	Maintenance Cost ($thousand/y)	Equipment Life (y)
A	20	7	4
B	25	5	6
C	40	3	7

51. The cumulative **discounted** cash flow diagram for a process is shown in Figure P10.51; all numbers are given in $million and have been discounted to time $t = 0$. The project life is ten years after startup and the construction period is two years. The hurdle rate is 5% p.a. and the tax rate is 40% p.a. The 5-year MACRS (with a ½ year convention) was used for the depreciation calculations. For this project, answer the following questions. The answers for parts (a–d) should be given in terms of the cost or value at the time of purchase, that is, **not** the discounted value:

What was the cost of land?

What was the fixed capital investment excluding land, FCI_L?

What was the cost for working capital (WC)?

What was the value for (R-COM_d)?

What is the discounted payback period ($DPBP$)?

What is the net present value of the project (NPV)?

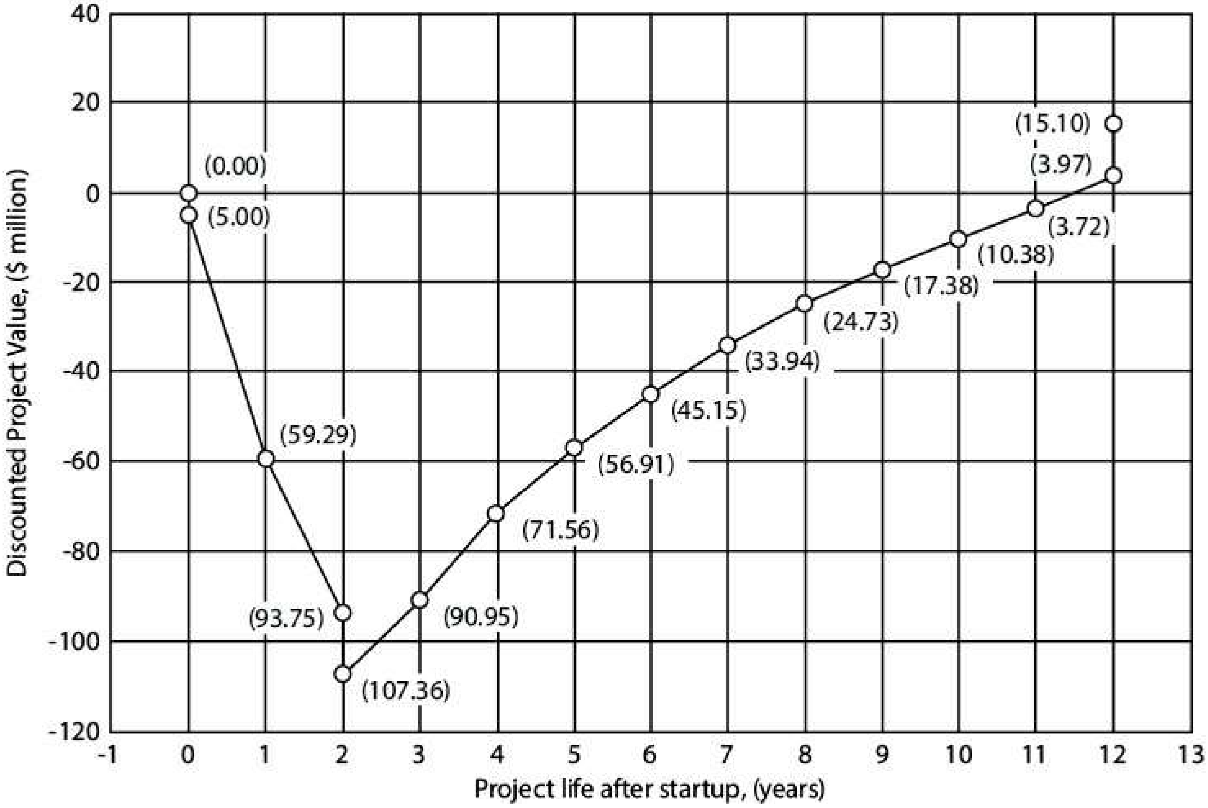

Figure P10.51 Cash Flow Diagram for Problem 51